Mplus Short Courses

Growth Modeling With Latent Variables Using Mplus

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Mplus Background

- Inefficient dissemination of statistical methods:
  - Many good methods contributions from biostatistics, psychometrics, etc are underutilized in practice
- Fragmented presentation of methods:
  - Technical descriptions in many different journals
  - Many different pieces of limited software
- Mplus: Integration of methods in one framework
  - Easy to use: Simple, non-technical language, graphics
  - Powerful: General modeling capabilities
- Mplus versions
  - V1: November 1998
  - V2: February 2001
  - V3: March 2004
  - V4: February 2006
- Mplus team: Linda & Bengt Muthén, Thuy Nguyen, Tihomir Asparouhov, Michelle Conn

Statistical Analysis With Latent Variables
A General Modeling Framework

Statistical Concepts Captured By Latent Variables

Continuous Latent Variables

- Measurement errors
- Factors
- Random effects
- Frailties, liabilities
- Variance components
- Missing data

Categorical Latent Variables

- Latent classes
- Clusters
- Finite mixtures
- Missing data

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Statistical Analysis With Latent Variables
A General Modeling Framework (Continued)

Models That Use Latent Variables

Continuous Latent Variables

- Factor analysis models
- Structural equation models
- Growth curve models
- Multilevel models

Categorical Latent Variables

- Latent class models
- Mixture models
- Discrete-time survival models
- Missing data models

Mplus integrates the statistical concepts captured by latent variables into a general modeling framework that includes not only all of the models listed above but also combinations and extensions of these models.

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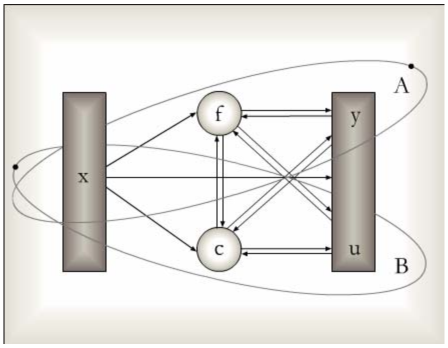
General Latent Variable Modeling Framework



- Observed variables
  - x background variables (no model structure)
  - y continuous and censored outcome variables
  - u categorical (dichotomous, ordinal, nominal) and count outcome variables
- Latent variables
  - f continuous variables
    - – interactions among f's
  - c categorical variables
    - – multiple c's

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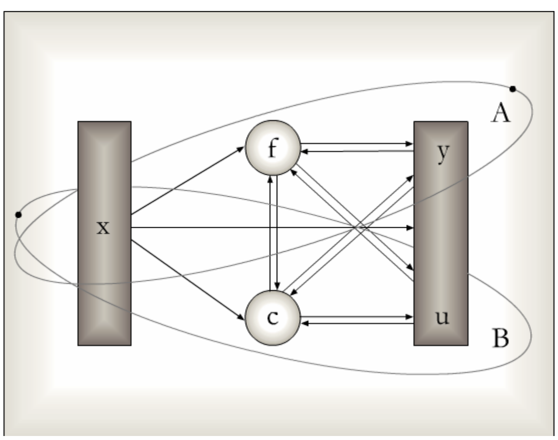
General Latent Variable Modeling Framework



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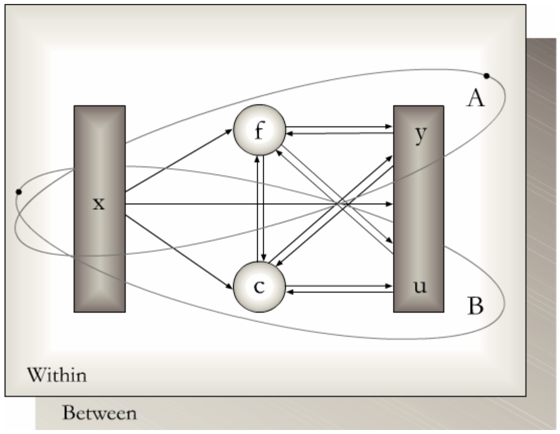
General Latent Variable Modeling Framework



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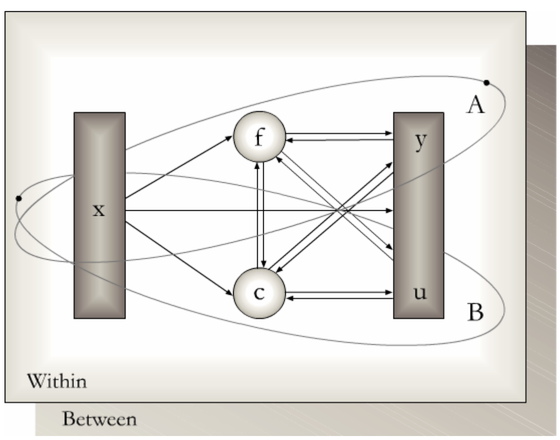
General Latent Variable Modeling Framework



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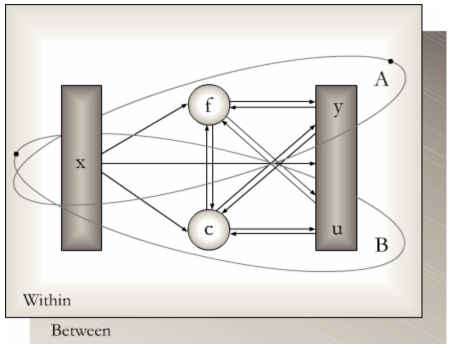
General Latent Variable Modeling Framework



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Overview

Single-Level Analysis

	Cross-Sectional	Longitudinal
Continuous Observed And Latent Variables	***Day 1*** Regression Analysis Path Analysis Exploratory Factor Analysis Confirmatory Factor Analysis Structural Equation Modeling	***Day 2*** Growth Analysis
Adding Categorical Observed And Latent Variables	***Day 3*** Regression Analysis Path Analysis Exploratory Factor Analysis Confirmatory Factor Analysis Structural Equation Modeling Latent Class Analysis Factor Mixture Analysis Structural Equation Mixture Modeling	***Day 4*** Latent Transition Analysis Latent Class Growth Analysis Growth Analysis Growth Mixture Modeling Discrete-Time Survival Mixture Analysis Missing Data Analysis

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Overview (Continued)

Multilevel Analysis

	Cross-Sectional	Longitudinal
Continuous Observed And Latent Variables	***Day 5*** Regression Analysis Path Analysis Exploratory Factor Analysis Confirmatory Factor Analysis Structural Equation Modeling	***Day 5*** Growth Analysis
Adding Categorical Observed And Latent Variables	***Day 5*** Latent Class Analysis Factor Mixture Analysis	***Day 5*** Growth Mixture Modeling

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Typical Examples Of Growth Modeling

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LSAY Data

Longitudinal Study of American Youth (LSAY)

- Two cohorts measured each year beginning in 1987
  - Cohort 1 - Grades 10, 11, and 12
  - Cohort 2 - Grades 7, 8, 9, and 10
- Each cohort contains approximately 60 schools with approximately 60 students per school
- Variables - math and science achievement items, math and science attitude measures, and background variables from parents, teachers, and school principals
- Approximately 60 items per test with partial item overlap across grades - adaptive tests

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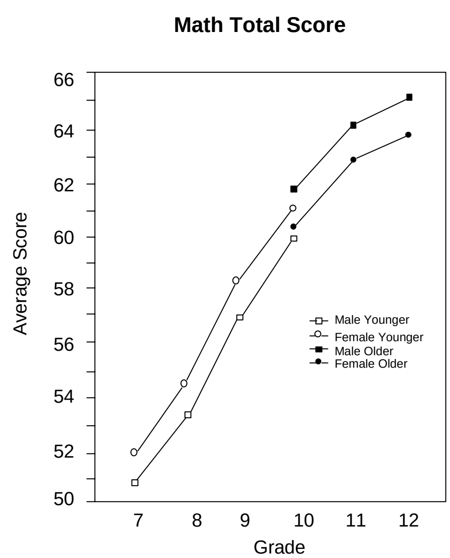

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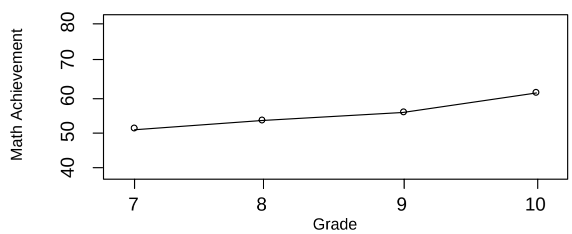
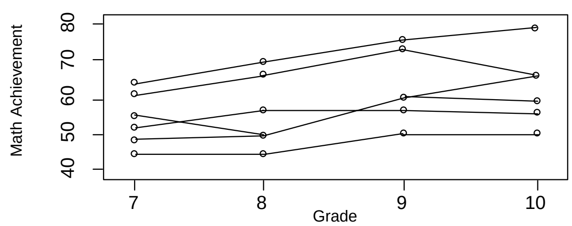
Mean Curve



Individual Curves



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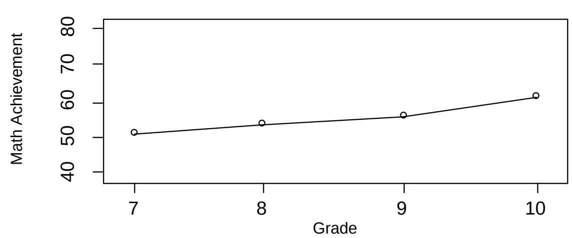
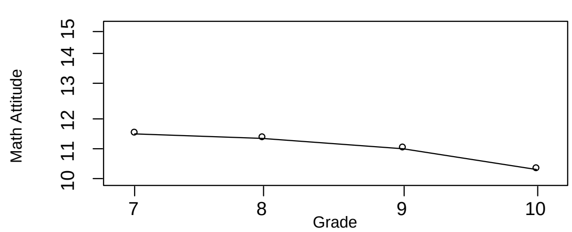
Sample Means for Math



Sample Means for Attitude Towards Math



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Maternal Health Project Data

Maternal Health Project (MHP)

- Mothers who drank at least three drinks a week during their first trimester plus a random sample of mothers who used alcohol less often
- Mothers measured at fourth month and seventh month of pregnancy, at delivery, and at 8, 18, and 36 months postpartum
- Offspring measured at 0, 8, 18 and 36 months
- Variables for mothers - demographic, lifestyle, current environment, medical history, maternal psychological status, alcohol use, tobacco use, marijuana use, other illicit drug use
- Variables for offspring - head circumference, height, weight, gestational age, gender, and ethnicity

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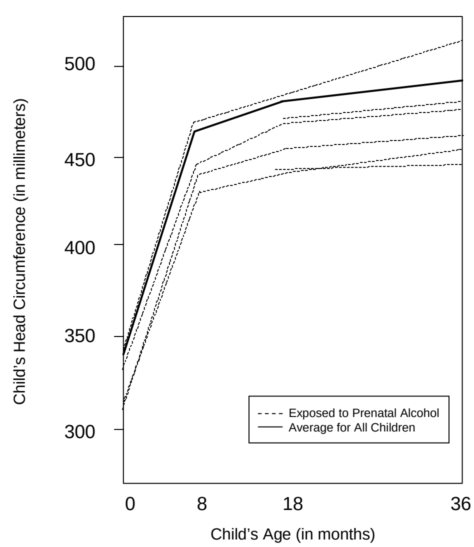
MHP: Offspring Head Circumference



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Basic Modeling Ideas

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Longitudinal Data: Three Approaches

Three modeling approaches for the regression of outcome on time (n is sample size, T is number of timepoints):

- **Use all n x T data points to do a single regression analysis**: Gives an intercept and a slope estimate for all individuals - does not account for individual differences or lack of independence of observations
- **Use each individual's T data points to do n regression analyses:** Gives an intercept and a slope estimate for each individual. Accounts for individual differences, but does not account for similarities among individuals
- **Use all n x T data points to do a single random effect regression analysis:** Gives an intercept and a slope estimate for each individual. Accounts for similarities among individuals by stipulating that all individuals' random effects come from a single, common population and models the non-independence of observations as show on the next page

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Individual Development Over Time

(1) $y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}$

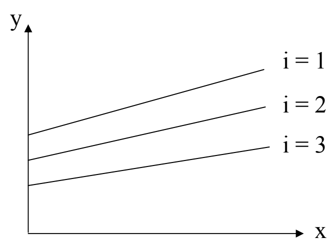
(2a) $\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}$

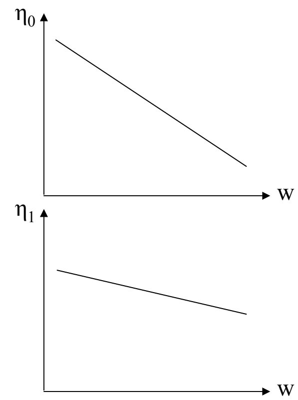
(2b) $\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}$

t = timepoint   i = individual   w = time-invariant covariate

y = outcome   x = time score

η_0 = intercept η_1 = slope





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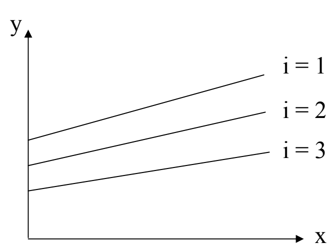
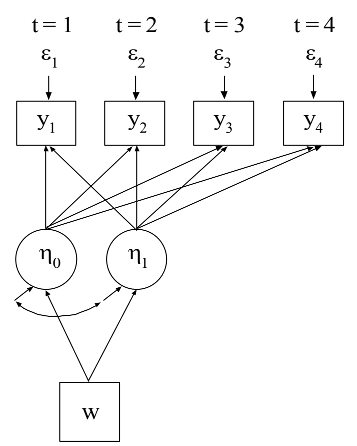
Individual Development Over Time





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Growth Modeling Frameworks

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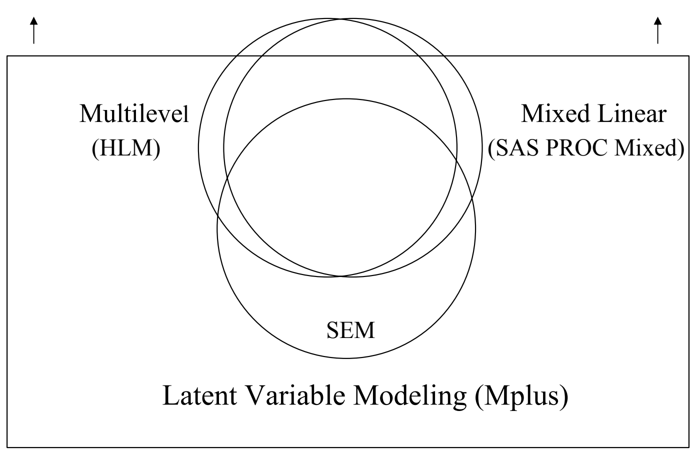
Growth Modeling Frameworks/Software



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Comparison Summary Of Multilevel, Mixed Linear, And SEM Growth Models

- Multilevel and mixed linear models are the same
- SEM differs from the multilevel and mixed linear models in two ways
  - Treatment of time scores
    - Time scores are data for multilevel and mixed linear models -- individuals can have different times of measurement
    - Time scores are parameters for SEM growth models -- time scores can be estimated
  - Treatment of time-varying covariates
    - Time-varying covariates have random effect coefficients for multilevel and mixed linear models -- coefficients vary over individuals
    - Time-varying covariates have fixed effect coefficients for SEM growth models -- coefficients vary over time

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Random Effects: Multilevel And Mixed Linear Modeling

Individual i ($i = 1, 2, \ldots, n$) observed at time point t ($t = 1, 2, \ldots T$).

Multilevel model with two levels (e.g. Raudenbush & Bryk, 2002, HLM).

- Level 1: $y_{ti} = \eta_{0i} + \eta_{1i}\, x_{ti} + \kappa_i\, w_{ti} + \varepsilon_{ti}$ (39)
- Level 2: $\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}$ (40)

  $\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}$ (41)

  $\kappa_i = \alpha + \gamma\, w_i + \zeta_i$ (42)

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Random Effects: Multilevel And Mixed Linear Modeling (Continued)

Mixed linear model:

$$y_{ti} = \textit{fixed part} + \textit{random part} \qquad (43)$$
$$= \alpha_0 + \gamma_0\, w_i + (\alpha_1 + \gamma_1\, w_i)\, x_{ti} + (\alpha + \gamma\, w_i)\, w_{ti} \qquad (44)$$
$$+ \zeta_{0i} + \zeta_{1i} x_{ti} + \zeta_i w_{ti} + \varepsilon_{ti}\,. \qquad (45)$$

E.g. "*time* x w_i" refers to γ_1 (e.g. Rao, 1958; Laird & Ware, 1982; Jennrich & Sluchter, 1986; Lindstrom & Bates, 1988; BMDP5V; Goldstein, 1995, MLn; SAS PROC MIXED - Littell et al. 1996 and Singer, 1999).

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Random Effects: SEM And Multilevel Modeling

SEM (Tucker, 1958; Meredith & Tisak, 1990; McArdle & Epstein 1987; SEM software):

Measurement part:

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \kappa_t\, w_{ti} + \varepsilon_{ti}\,. \qquad (46)$$

Compare with level 1 of multilevel:

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_{ti} + \kappa_i\, w_{ti} + \varepsilon_{ti}\,. \qquad (47)$$

Multilevel approach:

- x_{ti} as data: Flexible individually-varying times of observation
- Slopes for time-varying covariates vary over individuals

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Random Effects: SEM And Multilevel Modeling (Continued)

SEM approach:

- x_t as parameters: Flexible growth function form
- Slopes for time-varying covariates vary over time points

Structural part (same as level 2, except for κ_t):

$$\eta_{0i} = \alpha_0 + \gamma_0 \, w_i + \zeta_{0i} \, , \qquad (48)$$

$$\eta_{1i} = \alpha_1 + \gamma_1 \, w_i + \zeta_{1i} \, , \qquad (49)$$

κ_t not involved (parameter).

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Random Effects: Mixed Linear Modeling And SEM

Mixed linear model in matrix form:

$$\boldsymbol{y}_i = (y_{1i}, y_{2i}, \ldots, y_{Ti})' \qquad (51)$$

$$= \boldsymbol{X}_i \, \boldsymbol{\alpha} + \boldsymbol{Z}_i \, \boldsymbol{b}_i + \boldsymbol{e}_i \, . \qquad (52)$$

Here, ***X, Z*** are design matrices with known values, $\boldsymbol{\alpha}$ contains fixed effects, and $\boldsymbol{b}$ contains random effects. Compare with (43) - (45).

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Random Effects: Mixed Linear Modeling And SEM (Continued)

SEM in matrix form:

$$y_i = \nu + \Lambda\,\eta_i + K\,x_i + \varepsilon_i\,, \qquad (53)$$

$$\eta_i = \alpha + B\,\eta_i + \Gamma\,x_i + \zeta_i\,. \qquad (54)$$

$$y_i = \textit{fixed part} + \textit{random part}$$

$$= \text{v} + \Lambda\,(I - B)^{-1}\,\alpha + \Lambda\,(I - B)^{-1}\,\Gamma\,x_i + K\,x_i$$

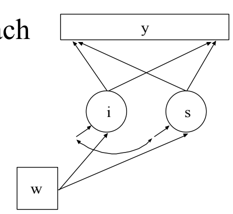
$$+ \Lambda\,(I - B)^{-1}\,\zeta_i + \varepsilon_i\,.$$

Assume $x_{ti} = x_t$, $\kappa_i = \kappa_t$ in (39). Then (39) is handled by (53) and (40) – (41) are handled by (54), putting x_t in Λ and w_{ti}, w_i in x_i.

Need for Λ_i, K_i, B_i, Γ_i.

Growth Modeling Approached In Two Ways: Data Arranged As Wide Versus Long

- Wide: Multivariate, Single-Level Approach

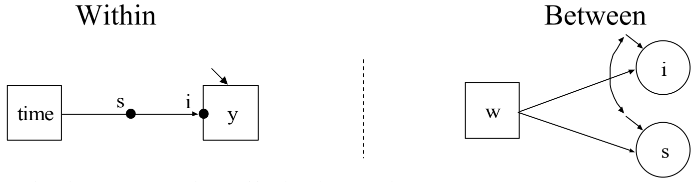


$$y_{ti} = i_i + s_i \times \text{time}_{ti} + \varepsilon_{ti}$$

i_i regressed on w_i
s_i regressed on w_i

- Long: Univariate, 2-Level Approach (CLUSTER = id)

Within Between



The intercept *i* is called *y* in Mplus

Multilevel Modeling In A Latent Variable Framework

Integrating multilevel and SEM analyses (Asparouhov & Muthén, 2002).

Flexible combination of random effects and other latent variables:

- Multilevel models with random effects (intercepts, slopes)
  - Individually-varying times of observation read as data
  - Random slopes for time-varying covariates
- SEM with factors on individual and cluster levels
- Models combining random effects and factors, e.g.
  - Cluster-level latent variable predictors with multiple indicators
  - Individual-level latent variable predictors with multiple indicators
- Special applications
  - Random coefficient regression (no clustering; heteroscedasticity)
  - Interactions between continuous latent variables and observed variables

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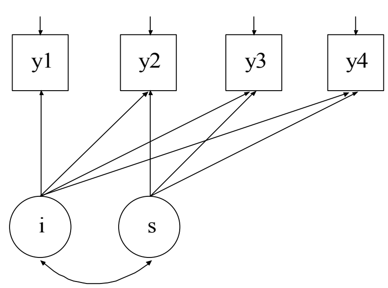
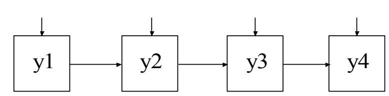
Alternative Models For Longitudinal Data

Growth Curve Model



Auto-Regressive Model



Hybrid Models

Curran & Bollen (2001)
McArdle & Hamagami (2001)

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Advantages Of Growth Modeling In A Latent Variable Framework

- Flexible curve shape
- Individually-varying times of observation
- Regressions among random effects
- Multiple processes
- Modeling of zeroes
- Multiple populations
- Multiple indicators
- Embedded growth models
- Categorical latent variables: growth mixtures

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The Latent Variable Growth Model In Practice

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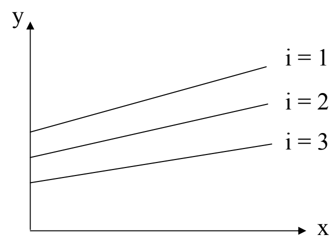
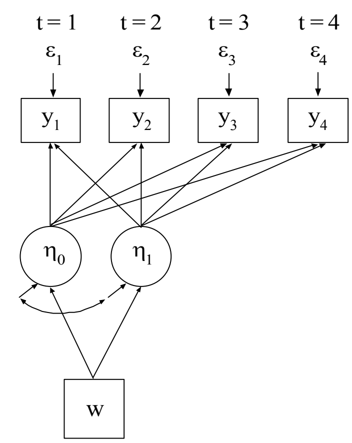
Individual Development Over Time





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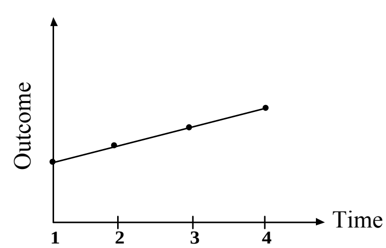
Specifying Time Scores For Linear Growth Models

Linear Growth Model

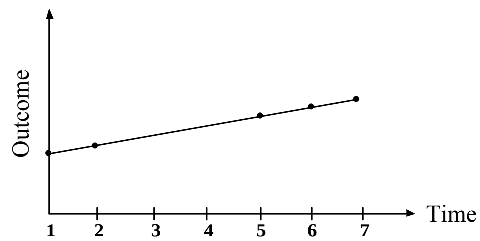
- Need two latent variables to describe a linear growth model: Intercept and slope



- Equidistant time scores for slope:

0	1	2	3
0	.1	.2	.3

Specifying Time Scores For Linear Growth Models (Continued)



- Nonequidistant time scores for slope:

0	1	4	5	6
0	.1	.4	.5	.6

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Interpretation Of The Linear Growth Factors

Model:

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}, \qquad (17)$$

where in the example $t = 1, 2, 3, 4$ and $x_t = 0, 1, 2, 3$:

$$y_{1i} = \eta_{0i} + \eta_{1i}\, 0 + \varepsilon_{1i}, \qquad (18)$$
$$\eta_{0i} = y_{1i} - \varepsilon_{1i}, \qquad (19)$$
$$y_{2i} = \eta_{0i} + \eta_{1i}\, 1 + \varepsilon_{2i}, \qquad (20)$$
$$y_{3i} = \eta_{0i} + \eta_{1i}\, 2 + \varepsilon_{3i}, \qquad (21)$$
$$y_{4i} = \eta_{0i} + \eta_{1i}\, 3 + \varepsilon_{4i}. \qquad (22)$$

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Interpretation Of The Linear Growth Factors (Continued)

Interpretation of the intercept growth factor
η_{0i} (initial status, level):
Systematic part of the variation in the outcome variable at the time point where the time score is zero.

- Unit factor loadings

Interpretation of the slope growth factor
η_{1i} (growth rate, trend):
Systematic part of the increase in the outcome variable for a time score increase of one unit.

- Time scores determined by the growth curve shape

Interpreting Growth Model Parameters

- Intercept Growth Factor Parameters
  - Mean
    - Average of the outcome over individuals at the timepoint with the time score of zero;
    - When the first time score is zero, it is the intercept of the average growth curve, also called initial status
  - Variance
    - Variance of the outcome over individuals at the timepoint with the time score of zero, excluding the residual variance

Interpreting Growth Model Parameters (Continued)

- Linear Slope Growth Factor Parameters
  - Mean – average growth rate over individuals
  - Variance – variance of the growth rate over individuals
  - Covariance with Intercept – relationship between individual intercept and slope values
- Outcome Parameters
  - Intercepts – not estimated in the growth model – fixed at zero to represent measurement invariance
  - Residual Variances – time-specific and measurement error variation
  - Residual Covariances – relationships between time-specific and measurement error sources of variation across time

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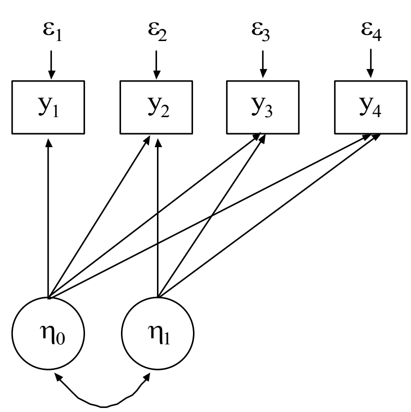
Latent Growth Model Parameters And Sources Of Model Misfit



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Latent Growth Model Parameters For Four Time Points

Linear growth over four time points, no covariates.

Free parameters in the H_1 unrestricted model:

- 4 means and 10 variances-covariances

Free parameters in the H_0 growth model:

(9 parameters, 5 d.f.):

- Means of intercept and slope growth factors
- Variances of intercept and slope growth factors
- Covariance of intercept and slope growth factors
- Residual variances for outcomes

Fixed parameters in the H_0 growth model:

- Intercepts of outcomes at zero
- Loadings for intercept growth factor at one
- Loadings for slope growth factor at time scores
- Residual covariances for outcomes at zero

Latent Growth Model Sources Of Misfit

Sources of misfit:

- Time scores for slope growth factor
- Residual covariances for outcomes
- Outcome variable intercepts
- Loadings for intercept growth factor

Model modifications:

- Recommended
  - Time scores for slope growth factor
  - Residual covariances for outcomes
- Not recommended
  - Outcome variable intercepts
  - Loadings for intercept growth factor

Latent Growth Model Parameters For Three Time Points

Linear growth over three time points, no covariates.

Free parameters in the H_1 unrestricted model:

- 3 means and 6 variances-covariances

Free parameters in the H_0 growth model

(8 parameters, 1 d.f.)

- Means of intercept and slope growth factors
- Variances of intercept and slope growth factors
- Covariance of intercept and slope growth factors
- Residual variances for outcomes

Fixed parameters in the H_0 growth model:

- Intercepts of outcomes at zero
- Loadings for intercept growth factor at one
- Loadings for slope growth factor at time scores
- Residual covariances for outcomes at zero

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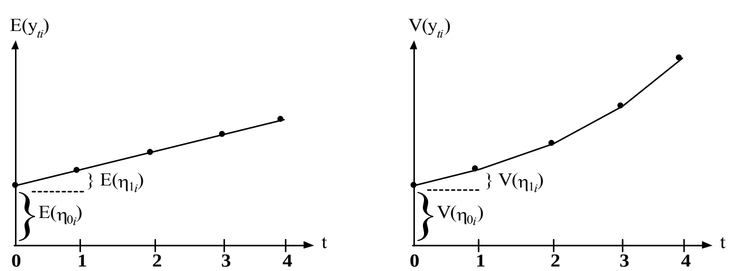
Growth Model Means And Variances

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,,$$
$$x_t = 0, 1, \ldots, T-1.$$

Expectation (mean; E) and variance (V):

$$E\,(y_{ti}) = E\,(\eta_{0i}\,) + E\,(\eta_{1i})\, x_t\,,$$
$$V\,(y_{ti}) = V\,(\eta_{0i}\,) + V\,(\eta_{1i})\, x_t^2 + 2x_t\, Cov\,(\eta_{0i}\,, \eta_{1i}) + V\,(\varepsilon_{ti})$$



V(ε_{ti}) constant over t
Cov(η_0, η_1) = 0

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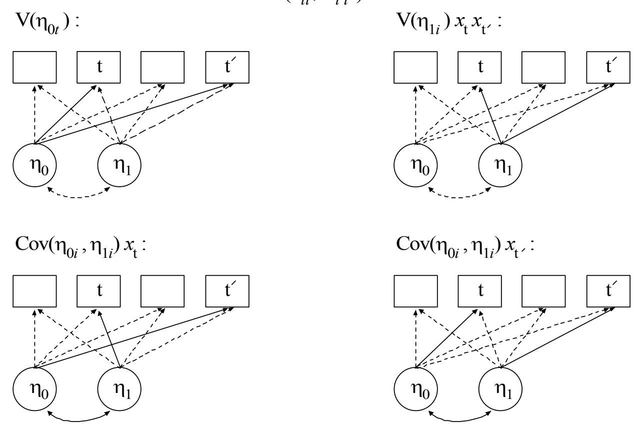
Growth Model Covariances

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti},$$
$$x_t = 0, 1, \ldots, T-1.$$

$$Cov(y_{ti}, y_{t'i}) = V(\eta_{0i}) + V(\eta_{1i})\, x_t\, x_{t'} + Cov(\eta_{0i}, \eta_{1i})\,(x_t + x_{t'}) + Cov(\varepsilon_{ti}, \varepsilon_{t'i}).$$



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Growth Model Estimation, Testing, And Model Modification

- Estimation: Model parameters
  - Maximum-likelihood (ML) estimation under normality
  - ML and non-normality robust s.e.'s
  - Quasi-ML (MUML): clustered data (multilevel)
  - WLS: categorical outcomes
  - ML-EM: missing data, mixtures
- Model Testing
  - Likelihood-ratio chi-square testing; robust chi square
  - Root mean square of approximation (RMSEA): Close fit ($\leq .05$)
- Model Modification
  - Expected drop in chi-square, EPC
- Estimation: Individual growth factor values (factor scores)
  - Regression method – Bayes modal – Empirical Bayes
  - Factor determinacy

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Alternative Growth Model Parameterizations

Parameterization 1 – for continuous outcomes

$$y_{ti} = \mathbf{0} + \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,, \qquad (32)$$

$$\eta_{0i} = \boldsymbol{\alpha}_0 + \zeta_{0i}\,, \qquad (33)$$

$$\eta_{1i} = \alpha_1 + \zeta_{1i}\,. \qquad (34)$$

Parameterization 2 – for categorical outcomes and multiple indicators

$$y_{ti} = \boldsymbol{\nu} + \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,, \qquad (35)$$

$$\eta_{0i} = \mathbf{0} + \zeta_{0i}\,, \qquad (36)$$

$$\eta_{1i} = \alpha_1 + \zeta_{1i}\,. \qquad (37)$$

Alternative Growth Model Parameterizations

Parameterization 1 – for continuous outcomes

- Outcome variable intercepts fixed at zero
- Growth factor means free to be estimated

```
MODEL:    i BY y1-y4@1;
          s BY y1@0 y2@1 y3@2 y4@3;
          [y1-y4@0 i s];
```

Parameterization 2 – for categorical outcomes and multiple indicators

- Outcome variable intercepts constrained to be equal
- Intercept growth factor mean fixed at zero

```
MODEL:    i BY y1-y4@1;
          s BY y1@0 y2@1 y3@2 y4@3;
          [y1-y4] (1);
          [i@0 s];
```

Simple Examples Of Growth Modeling

55

Steps In Growth Modeling

- Preliminary descriptive studies of the data: means, variances, correlations, univariate and bivariate distributions, outliers, etc.
- Determine the shape of the growth curve from theory and/or data
    - Individual plots
    - Mean plot
- Consider change in variance across time
- Fit model without covariates using fixed time scores
- Modify model as needed
- Add covariates

56

LSAY Data

The data come from the Longitudinal Study of American Youth (LSAY). Two cohorts were measured at four time points beginning in 1987. Cohort 1 was measured in Grades 10, 11, and 12. Cohort 2 was measured in Grades 7, 8, 9, and 10. Each cohort contains approximately 60 schools with approximately 60 students per school. The variables measured include math and science achievement items, math and science attitude measures, and background information from parents, teachers, and school principals. There are approximately 60 items per test with partial item overlap across grades – adaptive tests.

Data for the analysis include the younger females. The variables include math achievement from Grades 7, 8, 9, and 10 and the background variables of mother's education and home resources.

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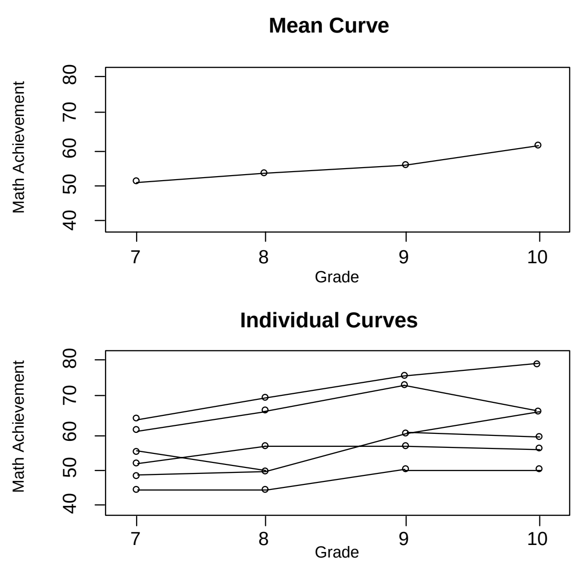


58

Input For LSAY TYPE=BASIC Analysis

```
TITLE:     LSAY For Younger Females With Listwise Deletion
           TYPE=BASIC Analysis
DATA:      FILE IS lsay.dat;
           FORMAT IS 3F8.0 F8.4 8F8.2 3F8.0;
VARIABLE:  NAMES ARE cohort id school weight math7 math8 math9
           math10 att7 att8 att9 att10 gender mothed homeres;
           USEOBS = (gender EQ 1 AND cohort EQ 2);
           MISSING = ALL (999);
           USEVAR = math7-math10;
ANALYSIS:  TYPE = BASIC;
PLOT:      TYPE = PLOT1;
```


Sample Statistics For LSAY Data

n = 984

Means

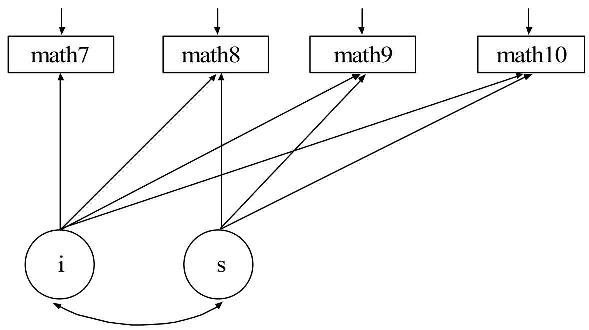
	MATH7	MATH8	MATH9	MATH10
	52.750	55.411	59.128	61.796

Covariances

	MATH7	MATH8	MATH9	MATH10
MATH7	81.107			
MATH8	67.663	82.829		
MATH9	73.150	76.513	100.986	
MATH10	77.952	82.668	95.158	131.326

Correlations

	MATH7	MATH8	MATH9	MATH10
MATH7	1.000			
MATH8	0.826	1.000		
MATH9	0.808	0.837	1.000	
MATH10	0.755	0.793	0.826	1.000



61

Input For LSAY Linear Growth Model Without Covariates

```
TITLE:     LSAY For Younger Females With Listwise Deletion
           Linear Growth Model Without Covariates
DATA:      FILE IS lsay.dat;
           FORMAT IS 3F8.0 F8.4 8F8.2 3F8.0;
VARIABLE:  NAMES ARE cohort id school weight math7 math8 math9
           math10 att7 att8 att9 att10 gender mothed homeres;
           USEOBS = (gender EQ 1 AND cohort EQ 2);
           MISSING = ALL (999);
           USEVAR = math7-math10;
ANALYSIS:  TYPE = MEANSTRUCTURE;
MODEL:     i BY math7-math10@1;
           s BY math7@0 math8@1 math9@2 math10@3;
           [math7-math10@0];
           [i s];
OUTPUT:    SAMPSTAT STANDARDIZED MODINDICES (3.84);
```

Alternative language:

MODEL: i s | math7@0 math8@1 math9@2 math10@3;

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Output Excerpts LSAY Linear Growth Model Without Covariates

Tests Of Model Fit

```
Chi-Square Test of Model Fit
          Value                         22.664
          Degrees of Freedom                 5
          P-Value                       0.0004

CFI/TLI
          CFI                           0.995
          TLI                           0.994

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                      0.060
          90 Percent C.I.               0.036   0.086
          Probability RMSEA <= .05      0.223

SRMR (Standardized Root Mean Square Residual)
          Value                         0.025
```

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Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

Modification Indices

	M.I.	E.P.C.	Std.E.P.C.	StdYX E.P.C.
S BY MATH7	6.793	0.185	0.254	0.029
S BY MATH8	14.694	-0.169	-0.233	-0.025
S BY MATH9	9.766	0.155	0.213	0.021

64

Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
I BY					
MATH7	1.000	.000	.000	8.029	.906
MATH8	1.000	.000	.000	8.029	.861
MATH9	1.000	.000	.000	8.029	.800
MATH10	1.000	.000	.000	8.029	.708
S BY					
MATH7	.000	.000	.000	.000	.000
MATH8	1.000	.000	.000	1.377	.148
MATH9	2.000	.000	.000	2.753	.274
MATH10	3.000	.000	.000	4.130	.364

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Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

Means					
I	52.623	.275	191.076	6.554	6.554
S	3.105	.075	41.210	2.255	2.255
Intercepts					
MATH7	.000	.000	.000	.000	.000
MATH8	.000	.000	.000	.000	.000
MATH9	.000	.000	.000	.000	.000
MATH10	.000	.000	.000	.000	.000

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Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

I WITH					
S	3.491	.730	4.780	.316	.316
Residual Variances					
MATH7	14.105	1.253	11.259	14.105	.180
MATH8	13.525	.866	15.610	13.525	.156
MATH9	14.726	.989	14.897	14.726	.146
MATH10	25.989	1.870	13.898	25.989	.202
Variances					
I	64.469	3.428	18.809	1.000	1.000
S	1.895	.322	5.894	1.000	1.000

R-Square

Observed Variable	R-Square
MATH7	0.820
MATH8	0.844
MATH9	0.854
MATH10	0.798

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Growth Model With Free Time Scores

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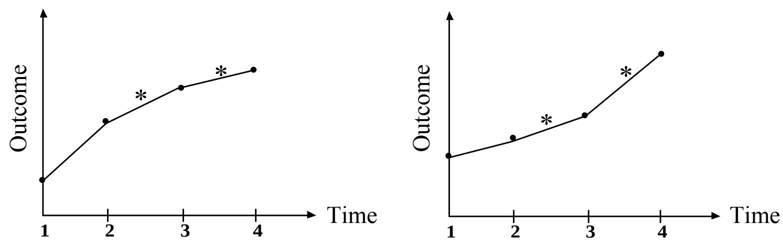
Specifying Time Scores For Non-Linear Growth Models With Estimated Time Scores

Non-linear growth models with estimated time scores

- Need two latent variables to describe a non-linear growth model: Intercept and slope



Time scores: 0 1 Estimated Estimated

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Interpretation Of Slope Growth Factor Mean For Non-Linear Models

- The slope growth factor mean is the change in the outcome variable for a one unit change in the time score
- In non-linear growth models, the time scores should be chosen so that a one unit change occurs between timepoints of substantive interest.
  - An example of 4 timepoints representing grades 7, 8, 9, and 10
    - Time scores of 0 1 * * – slope factor mean refers to change between grades 7 and 8
    - Time scores of 0 * * 1 – slope factor mean refers to change between grades 7 and 10

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Growth Model With Free Time Scores

- Identification of the model – for a model with two growth factors, at least one time score must be fixed to a non-zero value (usually one) in addition to the time score that is fixed at zero (centering point)
- Interpretation—cannot interpret the mean of the slope growth factor as a constant rate of change over all timepoints, but as the rate of change for a time score change of one.
- Approach—fix the time score following the centering point at one
- Choice of time score starting values if needed
  - Means 52.75 55.41 59.13 61.80
  - Differences 2.66 3.72 2.67
  - Time scores 0 1 >2 >2+1

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Input Excerpts For LSAY Linear Growth Model With Free Time Scores Without Covariates

```
MODEL:     i s | math7@0 math8@1 math9 math10;

OUTPUT:    RESIDUAL;


Alternative language:

MODEL:     i BY math7-math10@1;
           s BY math7@0 math8@1 math9 math10;
           [math7-math10@0];
           [i s];
```

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Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates

n = 984

Tests Of Model Fit

Chi-Square Test of Model Fit			
	Value	4.222	
	Degrees of Freedom	3	
	P-Value	0.2373	
CFI/TLI			
	CFI	1.000	
	TLI	0.999	
RMSEA (Root Mean Square Error Of Approximation)			
	Estimate	0.020	
	90 Percent C.I.	0.000	0.061
	Probability RMSEA <= .05	0.864	
SRMR (Standardized Root Mean Square Residual)			
	Value	0.015	

Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

Selected Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
I |					
MATH7	1.000	.000	.000	8.029	.903
MATH8	1.000	.000	.000	8.029	.870
MATH9	1.000	.000	.000	8.029	.797
MATH10	1.000	.000	.000	8.029	.708
S |					
MATH7	.000	.000	.000	.000	.000
MATH8	1.000	.000	.000	1.134	.123
MATH9	**2.452**	**.133**	**18.442**	**2.780**	**.276**
MATH10	**3.497**	**.199**	**17.540**	**3.966**	**.350**

Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
S WITH					
I	3.110	.600	5.186	.342	.342
Variances					
I	64.470	3.394	18.994	1.000	1.000
S	**1.286**	**.265**	**4.853**	**1.000**	**1.000**
Means					
I	52.785	.283	186.605	6.574	6.574
S	**2.586**	**.167**	**15.486**	**2.280**	**2.280**

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Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

Residuals

Model Estimated Means/Intercepts/Thresholds

MATH7	MATH8	MATH9	MATH10
52.785	55.370	59.123	61.827

Residuals for Means/Intercepts/Thresholds

MATH7	MATH8	MATH9	MATH10
-.035	.041	.004	-.031

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Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

Model Estimated Covariances/Correlations/Residual Correlations

	MATH7	MATH8	MATH9	MATH10
MATH7	79.025			
MATH8	67.580	85.180		
MATH9	72.094	78.356	101.588	
MATH10	75.346	82.952	93.994	128.477

Residuals for Covariances/Correlations/Residual Correlations

	MATH7	MATH8	MATH9	MATH10
MATH7	1.999			
MATH8	.014	-2.436		
MATH9	.981	-1.921	-.705	
MATH10	2.527	-.368	1.067	2.715

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Covariates In The Growth Model

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Covariates In The Growth Model

- Types of covariates
  - Time-invariant covariates—vary across individuals not time, explain the variation in the growth factors
  - Time-varying covariates—vary across individuals and time, explain the variation in the outcomes beyond the growth factors

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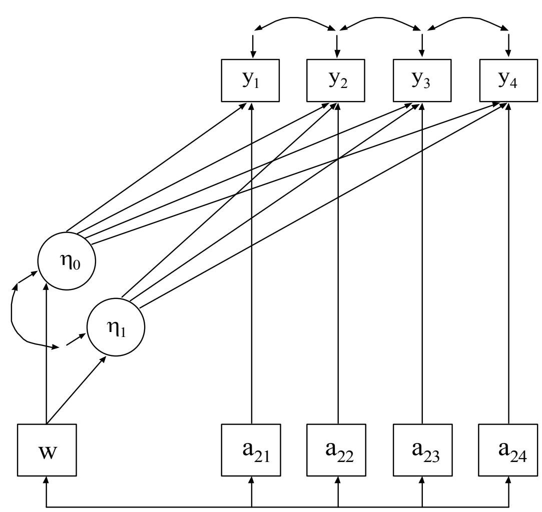
Time-Invariant And Time-Varying Covariates



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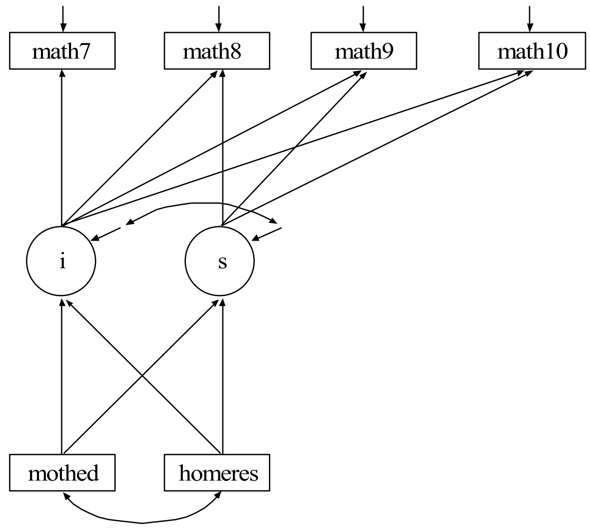
LSAY Growth Model With Time-Invariant Covariates



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Input Excerpts For LSAY Linear Growth Model With Free Time Scores And Covariates

```
VARIABLE:  NAMES ARE cohort id school weight math7 math8 math9
           math10 att7 att8 att9 att10 gender mothed homeres;
           USEOBS = (gender EQ 1 AND cohort EQ 2);
           MISSING = ALL (999);
           USEVAR = math7-math10 mothed homeres;
ANALYSIS:  !ESTIMATOR = MLM;
MODEL:     i s | math7@0 math8@1 math9 math10;
           i s ON mothed homeres;
```

Alternative language:

```
MODEL:     i BY math7-math10@1;
           s BY math7@0 math8@1 math9 math10;
           [math7-math10@0];
           [i s];
           i s ON mothed homeres;
```

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Output Excerpts LSAY Growth Model With Free Time Scores And Covariates

n = 935

Tests Of Model Fit for ML

```
Chi-Square Test of Model Fit
             Value                              15.845
             Degrees of Freedom                      7
             P-Value                            0.0265
CFI/TLI
             CFI                                 0.998
             TLI                                 0.995
RMSEA (Root Mean Square Error Of Approximation)
             Estimate                            0.037
             90 Percent C.I.                     0.012  0.061
             Probability RMSEA <= .05            0.794
SRMR (Standardized Root Mean Square Residual)
             Value                               0.015
```

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Output Excerpts LSAY Growth Model With Free Time Scores And Covariates (Continued)

Tests Of Model Fit for MLM

```
Chi-Square Test of Model Fit
             Value                               8.554*
             Degrees of Freedom                      7
             P-Value                            0.2862
             Scaling Correction Factor           1.852
                 for MLM
CFI/TLI
             CFI                                 0.999
             TLI                                 0.999
RMSEA (Root Mean Square Error Of Approximation)
             Estimate                            0.015
SRMR (Standardized Root Mean Square Residual)
             Value                               0.015
WRMR (Weighted Root Mean Square Residual)
             Value                               0.567
```

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Output Excerpts LSAY Growth Model With Free Time Scores And Covariates (Continued)

Selected Estimates For ML

	Estimates	S.E.	Est./S.E.	Std	StdYX
I ON					
MOTHED	**2.054**	**.281**	**7.322**	**.257**	**.247**
HOMERES	**1.376**	**.182**	**7.546**	**.172**	**.255**
S ON					
MOTHED	.103	.068	1.524	.094	.090
HOMERES	**.149**	**.045**	**3.334**	**.136**	**.201**
I WITH					
S	2.604	.559	4.658	.297	.297
Residual Variances					
I	53.931	2.995	18.008	.842	.842
S	1.134	.253	4.488	.942	.942
Intercepts					
I	43.877	.790	55.531	5.484	5.484
S	1.859	.221	8.398	1.695	1.695

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Output Excerpts LSAY Growth Model With Free Time Scores And Covariates (Continued)

R-Square

Observed Variable	R-Square
MATH7	0.813
MATH8	0.849
MATH9	0.861
MATH10	0.796

Latent Variable	R-Square
I	.158
S	.058

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Model Estimated Average And Individual Growth Curves With Covariates

Model:

$$y_{ti} = \eta_{0i} + \eta_{1i} x_t + \varepsilon_{ti} , \quad (23)$$
$$\eta_{0i} = \alpha_0 + \gamma_0 w_i + \zeta_{0i} , \quad (24)$$
$$\eta_{1i} = \alpha_1 + \gamma_1 w_i + \zeta_{1i} , \quad (25)$$

Estimated growth factor means:

$$\hat{E}(\eta_{0i}) = \hat{\alpha}_0 + \hat{\gamma}_0 \overline{w} , \quad (26)$$
$$\hat{E}(\eta_{1i}) = \hat{\alpha}_1 + \hat{\gamma}_1 \overline{w} . \quad (27)$$

Estimated outcome means:

$$\hat{E}(y_{ti}) = \hat{E}(\eta_{0i}) + \hat{E}(\eta_{1i}) x_t . \quad (28)$$

Estimated outcomes for individual *i* :

$$\hat{y}_{ti} = \hat{\eta}_{0i} + \hat{\eta}_{1i} x_t \quad (29)$$

where $\hat{\eta}_{0i}$ and $\hat{\eta}_{1i}$ are estimated factor scores. $\hat{y}_{ti}$ can be used for prediction purposes.

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Model Estimated Means With Covariates

Model estimated means are available using the TECH4 and RESIDUAL options of the OUTPUT command.

Estimated Intercept Mean = Estimated Intercept +
Estimated Slope (Mothed)*
Sample Mean (Mothed) +
Estimated Slope (Homeres)*
Sample Mean (Homeres)

43.88 + 2.05*2.31 + 1.38*3.11 = 52.9

Estimated Slope Mean = Estimated Intercept +
Estimated Slope (Mothed)*
Sample Mean (Mothed) +
Estimated Slope (Homeres)*
Sample Mean (Homeres)

1.86 + .10*2.31 + .15*3.11 = 2.56

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Model Estimated Means With Covariates (Continued)

Estimated Outcome Mean at Timepoint t =

Estimated Intercept Mean +
Estimated Slope Mean * (Time Score at Timepoint t)

Estimated Outcome Mean at Timepoint 1 =
52.9 + 2.56 * (0) = **52.9**
Estimated Outcome Mean at Timepoint 2 =
52.9 + 2.56 * (1.00) = **55.46**
Estimated Outcome Mean at Timepoint 3 =
52.9 + 2.56 * (2.45) = **59.17**
Estimated Outcome Mean at Timepoint 4 =
52.9 + 2.56 * (3.50) = **61.86**

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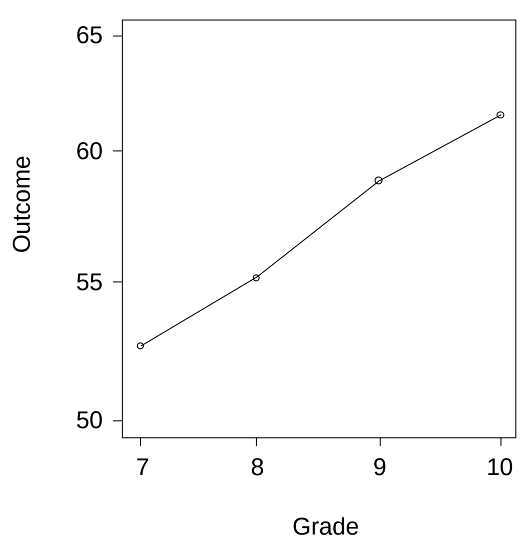
Estimated LSAY Curve



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Centering

Centering

- Centering determines the interpretation of the intercept growth factor
- The centering point is the timepoint at which the time score is zero
- A model can be estimated for different centering points depending on which interpretation is of interest
- Models with different centering points give the same model fit because they are reparameterizations of the model
- Changing the centering point in a linear growth model with four timepoints

Timepoints	1	2	3	4	Centering at
Time scores	0	1	2	3	Timepoint 1
	-1	0	1	2	Timepoint 2
	-2	-1	0	1	Timepoint 3
	-3	-2	-1	0	Timepoint 4

Input Excerpts For LSAY Growth Model With Free Time Scores And Covariates Centered At Grade 10

```
MODEL:     i s | math7*-3 math8*-2 math9@-1 math10@0;
           i s ON mothed homeres;


Alternative language:

MODEL:     i BY math7-math10@1;
           s BY math7*-3 math8*-2 math9@-1 math10@0;
           [math7-math10@0];
           [i s];
           i s ON mothed homeres;
```

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Output Excerpts LSAY Growth Model With Free Time Scores And Covariates Centered At Grade 10

n = 935

Tests of Model Fit

```
CHI-SQUARE TEST OF MODEL FIT

          Value                         15.845
          Degrees of Freedom                 7
          P-Value                        .0265

RMSEA (ROOT MEAN SQUARE ERROR OF APPROXIMATION)

          Estimate                       .037
          90 Percent C.I.                .012    .061
          Probability RMSEA <= .05       .794
```

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Output Excerpts LSAY Growth Model With Free Time Scores And Covariates Centered At Grade 10 (Continued)

Selected Estimates

		Estimates	S.E.	Est./S.E.	Std	StdYX
I	ON					
	MOTHED	2.418	0.353	6.851	0.238	0.229
	HOMERES	1.903	0.229	8.294	0.187	0.277
S	ON					
	MOTHED	0.111	0.073	1.521	0.094	0.090
	HOMERES	0.161	0.049	3.311	0.136	0.201

95

Further Readings On Introductory Growth Modeling

Bijleveld, C. C. J. H., & van der Kamp, T. (1998). Longitudinal data analysis: Designs, models, and methods. Newbury Park: Sage.

Bollen, K.A. & Curran, P.J. (2006). Latent curve models. A structural equation perspective. New York: Wiley.

Muthén, B. & Khoo, S.T. (1998). Longitudinal studies of achievement growth using latent variable modeling. Learning and Individual Differences, Special issue: latent growth curve analysis, 10, 73-101. (#80)

Muthén, B. & Muthén, L. (2000). The development of heavy drinking and alcohol-related problems from ages 18 to 37 in a U.S. national sample. Journal of Studies on Alcohol, 61, 290-300. (#83)

Raudenbush, S.W. & Bryk, A.S. (2002). Hierarchical linear models: Applications and data analysis methods. Second edition. Newbury Park, CA: Sage Publications.

Singer, J.D. & Willett, J.B. (2003). Applied longitudinal data analysis. Modeling change and event occurrence. New York, NY: Oxford University Press.

Snijders, T. & Bosker, R. (1999). Multilevel analysis. An introduction to basic and advanced multilevel modeling. Thousand Oakes, CA: Sage Publications.

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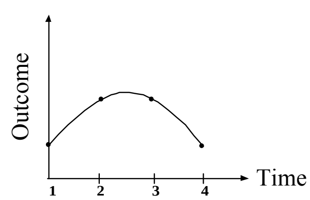
Further Practical Issues

Specifying Time Scores For Quadratic Growth Models

Quadratic growth model

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \eta_{2i}\ x_t^2 + \varepsilon_{ti}$$

- Need three latent variables to describe a quadratic growth model: Intercept, linear slope, quadratic slope



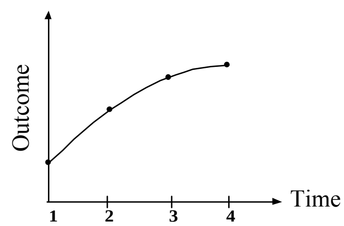
- Linear slope time scores: 0 1 2 3
  0 .1 .2 .3
- Quadratic slope time scores: 0 1 4 9
  0 .01 .04 .09

Specifying Time Scores For Non-Linear Growth Models With Fixed Time Scores

Non-Linear Growth Models with Fixed Time scores

- Need two latent variables to describe a non-linear growth model: Intercept and slope

Growth model with a logarithmic growth curve--ln(t)

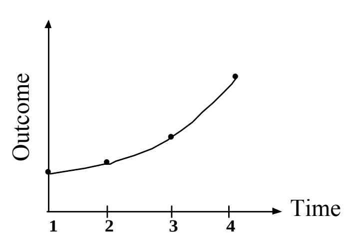


Time scores: 0 0.69 1.10 1.39

99

Specifying Time Scores For Non-Linear Growth Models With Fixed Time Scores (Continued)

Growth model with an exponential growth curve–
exp(t-1) - 1



Time scores: 0 1.72 6.39 19.09

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Piecewise Growth Modeling

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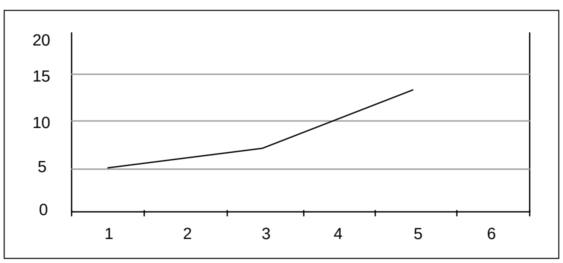
Piecewise Growth Modeling

- Can be used to represent different phases of development
- Can be used to capture non-linear growth
- Each piece has its own growth factor(s)
- Each piece can have its own coefficients for covariates

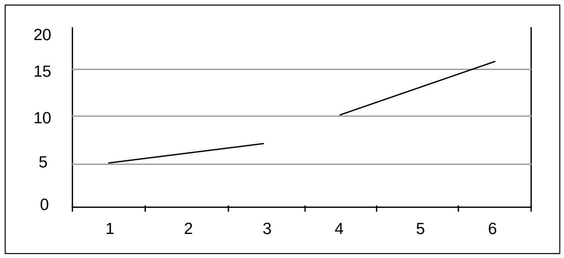


One intercept growth factor, two slope growth factors

0	1	2	2	2	2	Time scores piece 1
0	0	0	1	2	3	Time scores piece 2

102

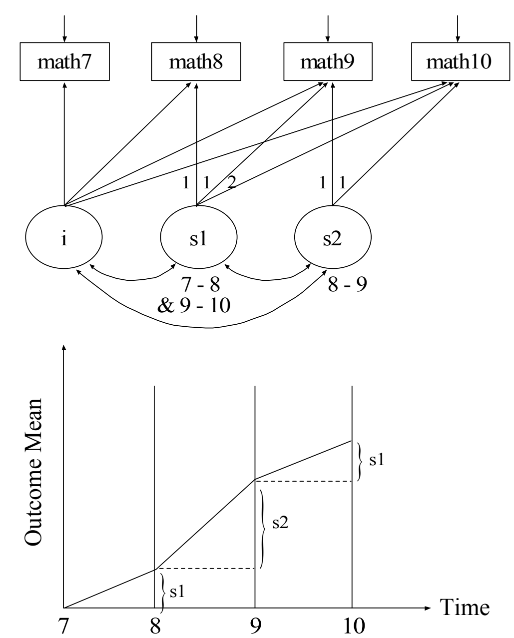
Piecewise Growth Modeling (Continued)



Two intercept growth factors, two slope growth factors

0	1	2				Time scores piece 1
		0	1	2		Time scores piece 2

103



104

Input For LSAY Piecewise Growth Model With Covariates

```
MODEL:     i s1 | math7@0 math8@1 math9@1 math10@2;
           i s2 | math7@0 math8@0 math9@1 math10@1;
           i s1 s2 ON mothed homeres;


Alternative language:


MODEL:     i BY math7-math10@1;
           s1 BY math7@0 math8@1 math9@1 math10@2;
           s2 BY math7@0 math8@0 math9@1 math10@1;
           [math7-math10@0];
           [i s1 s2];
           i s1 s2 ON mothed homeres;
```

105

Output Excerpts LSAY Piecewise Growth Model With Covariates

n = 935

Tests of Model Fit

```
CHI-SQUARE TEST OF MODEL FIT

          Value                          11.721
          Degrees of Freedom                  3
          P-Value                         .0083

RMSEA (ROOT MEAN SQUARE ERROR OF APPROXIMATION)

          Estimate                         .056
          90 Percent C.I.                  .025   .091
          Probability RMSEA <= .05         .331
```

106

Output Excerpts LSAY Piecewise Growth Model With Covariates (Continued)

Selected Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
I ON					
MOTHED	2.127	.284	7.488	.266	.256
HOMERES	1.389	.185	7.524	.174	.257
S1 ON					
MOTHED	-.126	.147	-.858	-.113	-.109
HOMERES	.091	.096	.950	.081	.120
S2 ON					
MOTHED	.436	.191	2.285	.185	.178
HOMERES	.289	.124	2.329	.123	.181

107

Growth Model With Individually-Varying Times Of Observation And Random Slopes For Time-Varying Covariates

108

Growth Modeling In Multilevel Terms

Time point t, individual i (two-level modeling, no clustering):

y_{ti} : repeated measures of the outcome, e.g. math achievement
a_{1ti} : time-related variable; e.g. grade 7-10
a_{2ti} : time-varying covariate, e.g. math course taking
x_i : time-invariant covariate, e.g. grade 7 expectations

Two-level analysis with individually-varying times of observation and random slopes for time-varying covariates:

$$Level\ 1: y_{ti} = \pi_{0i} + \pi_{1i}\, a_{1ti} + \pi_{2ti}\, a_{2ti} + e_{ti}, \quad (55)$$

$$Level\ 2: \begin{cases} \pi_{0i} = \beta_{00} + \beta_{01}\, x_i + r_{0i}, \\ \pi_{1i} = \beta_{10} + \beta_{11}\, x_i + r_{1i}, \\ \pi_{2i} = \beta_{20} + \beta_{21}\, x_i + r_{2i}. \end{cases} \quad (56)$$

109

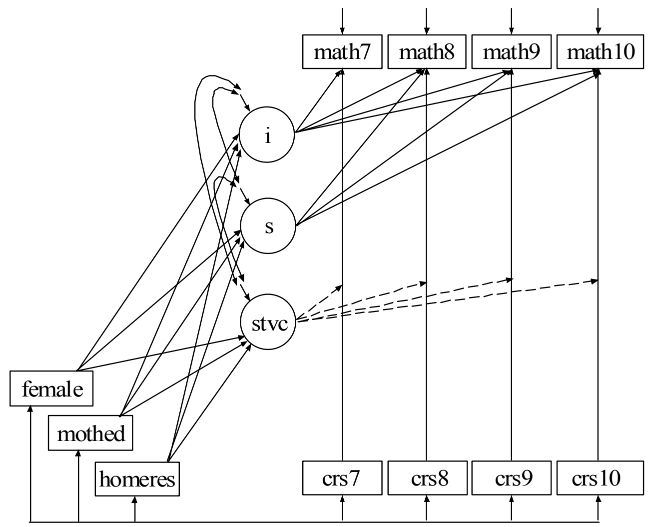
Growth Modeling In Multilevel Terms (Continued)

Time scores a_{1ti} read in as data (not loading parameters).

- π_{2ti} possible with time-varying random slope variances
- Flexible correlation structure for $V(e) = \Theta$ (T x T)
- Regressions among random coefficients possible, e.g.

$$\pi_{1i} = \beta_{10} + \gamma_1\, \pi_{0i} + \beta_{11}\, x_i + r_{1i}, \quad (57)$$
$$\pi_{2i} = \beta_{20} + \gamma_2\, \pi_{0i} + \beta_{21}\, x_i + r_{2i}. \quad (58)$$

110



Input For Growth Model With Individually Varying Times Of Observation

```
TITLE:      Growth model with individually varying times of
            observation and random slopes
DATA:       FILE IS lsaynew.dat;
            FORMAT IS 3F8.0 F8.4 8F8.2 3F8.0;
VARIABLE:   NAMES ARE math7 math8 math9 math10 crs7 crs8 crs9
            crs10 female mothed homeres a7-a10;

            ! crs7-crs10 = highest math course taken during each
            ! grade (0=no course, 1=low, basic, 2=average, 3=high.
            ! 4=pre-algebra, 5=algebra I, 6=geometry,
            ! 7=algebra II, 8=pre-calc, 9=calculus)

            MISSING ARE ALL (9999);
            CENTER = GRANDMEAN (crs7-crs10 mothed homeres);
            TSCORES = a7-a10;
```

Input For Growth Model With Individually Varying Times Of Observation (Continued)

```
DEFINE:      math7 = math7/10;
             math8 = math8/10;
             math9 = math9/10;
             math10 = math10/10;
ANALYSIS:    TYPE = RANDOM MISSING;
             ESTIMATOR = ML;
             MCONVERGENCE = .001;
MODEL:       i s | math7-math10 AT a7-a10;
             stvc | math7 ON crs7;
             stvc | math8 ON crs8;
             stvc | math9 ON crs9;
             stvc | math10 ON crs10;
             i ON female mothed homeres;
             s ON female mothed homeres;
             stvc ON female mothed homeres;
             i WITH s;
             stvc WITH i;
             stvc WITH s;
OUTPUT:      TECH8;
```

113

Output Excerpts For Growth Model With Individually Varying Times Of Observation And Random Slopes For Time-Varying Covariates

```
n = 2271
```

Tests of Model Fit

```
Loglikelihood

          H0 Value                      -8199.311

Information Criteria

          Number of Free Parameters           22
          Akaike (AIC)                  16442.623
          Bayesian (BIC)                16568.638
          Sample-Size Adjusted BIC      16498.740
             (n* = (n + 2) / 24)
```

114

Output Excerpts For Growth Model With Individually Varying Times Of Observation And Random Slopes For Time-Varying Covariates (Continued)

Model Results		Estimates	S.E.	Est./S.E.
I	ON			
	FEMALE	0.187	0.036	5.247
	MOTHED	0.187	0.018	10.231
	HOMERES	0.159	0.011	14.194
S	ON			
	FEMALE	-0.025	0.012	-2.017
	MOTHED	0.015	0.006	2.429
	HOMERES	0.019	0.004	4.835
STVC	ON			
	FEMALE	-0.008	0.013	-0.590
	MOTHED	0.003	0.007	0.429
	HOMERES	0.009	0.004	2.167
I	WITH			
	S	0.038	0.006	6.445
STVC	WITH			
	I	0.011	0.005	2.087
	S	0.004	0.002	2.033

115

Output Excerpts For Growth Model With Individually Varying Times Of Observation And Random Slopes For Time-Varying Covariates (Continued)

Intercepts			
MATH7	0.000	0.000	0.000
MATH8	0.000	0.000	0.000
MATH9	0.000	0.000	0.000
MATH10	0.000	0.000	0.000
I	4.992	0.025	198.456
S	0.417	0.009	47.275
STVC	0.113	0.010	11.416
Residual Variances			
MATH7	0.185	0.011	16.464
MATH8	0.178	0.008	22.232
MATH9	0.156	0.008	18.497
MATH10	0.169	0.014	12.500
I	0.570	0.023	25.087
S	0.036	0.003	12.064
STVC	0.012	0.002	5.055

116

Random Slopes

- In single-level modeling random slopes β_i describe variation across individuals *i*,

$$y_i = \alpha_i + \beta_i x_i + \varepsilon_i, \quad (100)$$
$$\alpha_i = \alpha + \zeta_{0i}, \quad (101)$$
$$\beta_i = \beta + \zeta_{1i}, \quad (102)$$

Resulting in heteroscedastic residual variances

$$V(y_i \mid x_i) = V(\beta_i)\ x_i^2 + \theta \quad (103)$$

- In two-level modeling random slopes β_j describe variation across clusters *j*

$$y_{ij} = a_j + \beta_j x_{ij} + \varepsilon_{ij}, \quad (104)$$
$$a_j = a + \zeta_{0j}, \quad (105)$$
$$\beta_j = \beta + \zeta_{1j}. \quad (106)$$

A small variance for a random slope typically leads to slow convergence of the ML-EM iterations. This suggests respecifying the slope as fixed.

Mplus allows random slopes for predictors that are

- Observed covariates
- Observed dependent variables
- Continuous latent variables

117

Computational Issues For Growth Models

- Decreasing variances of the observed variables over time may make the modeling more difficult
- Scale of observed variables – keep on a similar scale
- Convergence – often related to starting values or the type of model being estimated
  - Program stops because maximum number of iterations has been reached
    - If no negative residual variances, either increase the number of iterations or use the preliminary parameter estimates as starting values
    - If there are large negative residual variances, try better starting values
  - Program stops before the maximum number of iterations has been reached
    - Check if variables are on a similar scale
    - Try new starting values
- Starting values – the most important parameters to give starting values to are residual variances and the intercept growth factor mean
- Convergence for models using the | symbol
  - Non-convergence may be caused by zero random slope variances which indicates that the slopes should be fixed rather than random

118

Advanced Growth Models

119

Regressions Among Random Effects

120

Regressions Among Random Effects

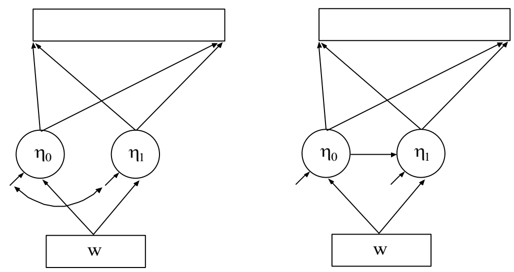
Standard multilevel model (where $x_t = 0, 1, \ldots, T$):

$$\text{Level 1: } y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}, \quad (1)$$
$$\text{Level 2a: } \eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}, \quad (2)$$
$$\text{Level 2b: } \eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}. \quad (3)$$

A useful type of model extension is to replace (3) by the regression equation

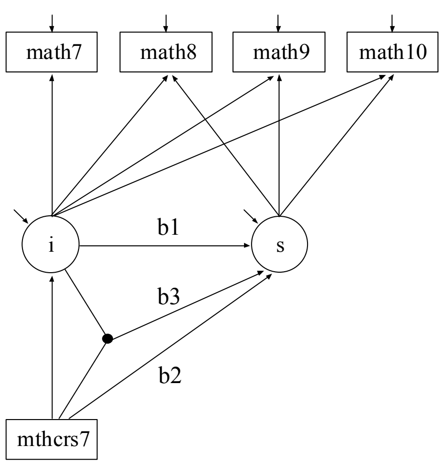
$$\eta_{1i} = \alpha + \beta\, \eta_{0i} + \gamma\, w_i + \zeta_i. \quad (4)$$

Example: Blood Pressure (Bloomqvist, 1977)



121

Growth Model With An Interaction



122

Input For A Growth Model With An Interaction Between A Latent And An Observed Variable

```
TITLE:     growth model with an interaction between a latent and an
           observed variable
DATA:      FILE IS lsay.dat;
VARIABLE:  NAMES ARE math7 math8 math9 math10 mthcrs7;
           MISSING ARE ALL (9999);
           CENTERING = GRANDMEAN (mthcrs7);
DEFINE:    math7 = math7/10;
           math8 = math8/10;
           math9 = math9/10;
           math10 = math10/10;
ANALYSIS:  TYPE=RANDOM MISSING;
MODEL:     i s | math7@0 math8@1 math9@2 math10@3;
           [math7-math10] (1);    !growth language defaults
           [i@0 s];               !overridden

           inter | i XWITH mthcrs7;
           s ON i mthcrs7 inter;
           i ON mthcrs7;
OUTPUT:    SAMPSTAT STANDARDIZED TECH1 TECH8;
```

123

Output Excerpts Growth Model With An Interaction Between A Latent And An Observed Variable

Tests Of Model Fit

```
Loglikelihood

              H0 Value                          -10068.944
Information Criteria

              Number of Free Parameters                 12
              Akaike (AIC)                       20161.887
              Bayesian (BIC)                     20234.365
              Sample-Size Adjusted BIC           20196.236
                 (n* = (n + 2) / 24)
```

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Output Excerpts Growth Model With An Interaction Between A Latent And An Observed Variable (Continued)

Model Results

```
                  Estimates      S.E.  Est./S.E.
 I        |
    MATH7            1.000      0.000     0.000
    MATH8            1.000      0.000     0.000
    MATH9            1.000      0.000     0.000
    MATH10           1.000      0.000     0.000
 S        |
    MATH7            0.000      0.000     0.000
    MATH8            1.000      0.000     0.000
    MATH9            2.000      0.000     0.000
    MATH10           3.000      0.000     0.000
```


Output Excerpts Growth Model With An Interaction Between A Latent And An Observed Variable (Continued)

```
                  Estimates       S.E.   Est./S.E.
 S        ON
    I                0.087       0.012      7.023
    INTER           -0.047       0.006     -7.301
 S        ON
    MTHCRS7          0.045       0.013      3.555
 I        ON
    MTHCRS7          0.632       0.016     40.412
```

Output Excerpts Growth Model With An Interaction Between A Latent And An Observed Variable (Continued)

	Estimates	S.E.	Est./S.E.
Intercepts			
MATH7	5.019	0.015	341.587
MATH8	5.019	0.015	341.587
MATH9	5.019	0.015	341.587
MATH10	5.019	0.015	341.587
I	0.000	0.000	0.000
S	0.417	0.007	57.749
Residual Variances			
MATH7	0.184	0.011	16.117
MATH8	0.178	0.009	20.109
MATH9	0.164	0.009	18.369
MATH10	0.173	0.015	11.509
I	0.528	0.018	28.935
S	0.037	0.004	10.027

Interpreting The Effect Of The Interaction Between Initial Status Of Growth In Math Achievement And Course Taking In Grade 6

- Model equation for slope s
  s = a + b1*i + b2*mthcrs7 + b3*i*mthcrs7 + e

  or, using a moderator function (Klein & Moosbrugger, 2000) where i moderates the influence of mthcrs7 on s
  s = a + b1*i + (b2 + b3*i)*mthcrs7 + e

- Estimated model

  Unstandardized
  s = 0.417 + 0.087*i + (0.045 – 0.047*i)*mthcrs7

  Standardized with respect to i and mthcrs7
  s = 0.42 + 0.08 * i + (0.04-0.04*i)*mthcrs7

Interpreting The Effect Of The Interaction Between Initial Status Of Growth In Math Achievement And Course Taking In Grade 6 (Continued)

- Interpretation of the standardized solution

  At the mean of i, which is zero, the slope increases 0.04 for 1 SD increase in mthcrs7

  At 1 SD below the mean of i, which is zero, the slope increases 0.08 for 1 SD increase in mthcrs7

  At 1 SD above the mean of i, which is zero, the slope does not increase as a function of mthcrs7

Growth Modeling With Parallel Processes

Growth Modeling With Parallel Processes

- Estimate a growth model for each process separately
  - Determine the shape of the growth curve
  - Fit model without covariates
  - Modify the model
- Joint analysis of both processes
- Add covariates

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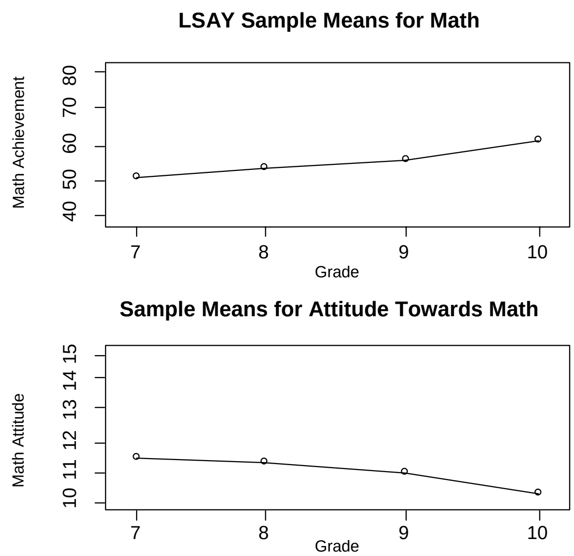
LSAY Data

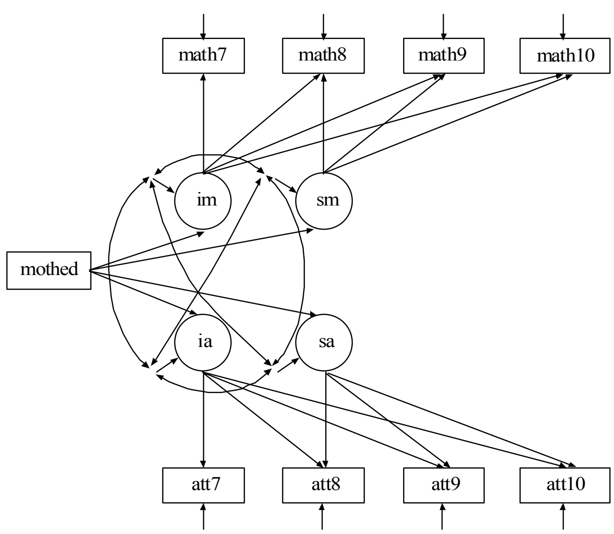
The data come from the Longitudinal Study of American Youth (LSAY). Two cohorts were measured at four time points beginning in 1987. Cohort 1 was measured in Grades 10, 11, and 12. Cohort 2 was measured in Grades 7, 8, 9, and 10. Each cohort contains approximately 60 schools with approximately 60 students per school. The variables measured include math and science achievement items, math and science attitude measures, and background information from parents, teachers, and school principals. There are approximately 60 items per test with partial item overlap across grades—adaptive tests.

Data for the analysis include the younger females. The variables include math achievement and math attitudes from Grades 7, 8, 9, and 10 and mother's education.

132


LSAY Sample Means for Math
Math Achievement
40 50 60 70 80
7 8 9 10
Grade
Sample Means for Attitude Towards Math
Math Attitude
10 11 12 13 14 15
7 8 9 10
Grade

133


math7
math8
math9
math10
im
sm
mothed
ia
sa
att7
att8
att9
att10

134

Input For LSAY Parallel Process Growth Model

```
TITLE:      LSAY For Younger Females With Listwise Deletion
            Parallel Process Growth Model-Math Achievement and
            Math Attitudes

DATA:       FILE IS lsay.dat;
            FORMAT IS 3f8 f8.4 8f8.2 3f8 2f8.2;

VARIABLE:   NAMES ARE cohort id school weight math7 math8 math9
            math10 att7 att8 att9 att10 gender mothed homeres
            ses3 sesq3;
            USEOBS = (gender EQ 1 AND cohort EQ 2);
            MISSING = ALL (999);
            USEVAR = math7-math10 att7-att10 mothed;

ANALYSIS:   TYPE = MEANSTRUCTURE;
```

135

Input For LSAY Parallel Process Growth Model

```
MODEL:      im sm | math7@0 math8@1 math9 math10;
            ia sa | att7@0 att8@1 att9@2 att10@3;
            im-sa ON mothed;

OUTPUT:     MODINDICES STANDARDIZED;
```

Alternative language:

```
            im BY math7-math10@1;
            sm BY math7@0 math8@1 math9 math10;

            ia BY att7-att10@1;
            sa BY att7@0 att8@1 att9@2 att10@3;

            [math7-math10@0 att7-att10@0];
            [im sm ia sa];

            im-sa ON mothed;
```

136

Output Excerpts LSAY Parallel Process Growth Model

n = 910

Tests of Model Fit

```
Chi-Square Test of Model Fit

          Value                         43.161
          Degrees of Freedom                24
          P-Value                        .0095

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                        .030
          90 Percent C.I.                 .015   .044
          Probability RMSEA <= .05        .992
```

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Output Excerpts LSAY Parallel Process Growth Model (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
IM ON					
MOTHED	**2.462**	**.280**	**8.798**	**.311**	**.303**
SM ON					
MOTHED	**.145**	**.066**	**2.195**	**.132**	**.129**
IA ON					
MOTHED	**.053**	**.086**	**.614**	**.025**	**.024**
SA ON					
MOTHED	**.012**	**.035**	**.346**	**.017**	**.017**

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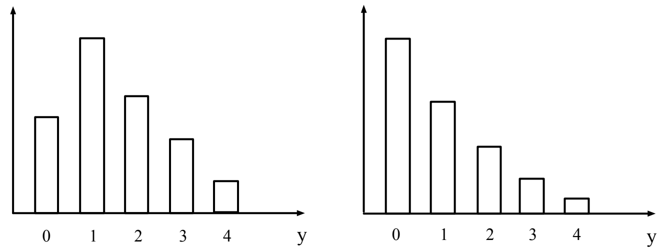
Output Excerpts LSAY Parallel Process Growth Model (Continued)

		Estimates	S.E.	Est./S.E.	Std	StdYX
SM	WITH					
IM		3.032	.580	5.224	.350	.350
IA	WITH					
IM		4.733	.702	6.738	.282	.282
SM		.544	.164	3.312	.235	.235
SA	WITH					
IM		-.276	.279	-.987	-.049	-.049
SM		.130	.066	1.976	.168	.168
IA		-.567	.115	-4.913	-.378	-.378

Modeling With Zeroes

Modeling With A Preponderance Of Zeros



- Outcomes: non-normal continuous – count – categorical
- Censored-normal modeling
- Two-part (semicontinuous modeling): Duan et al. (1983), Olsen & Schafer (2001)
- Mixture models, e.g. zero-inflated (mixture) Poisson (Roeder et al., 1999), censored-inflated, mover-stayer latent transition models, growth mixture models
- Onset (survival) followed by growth: Albert & Shih (2003)

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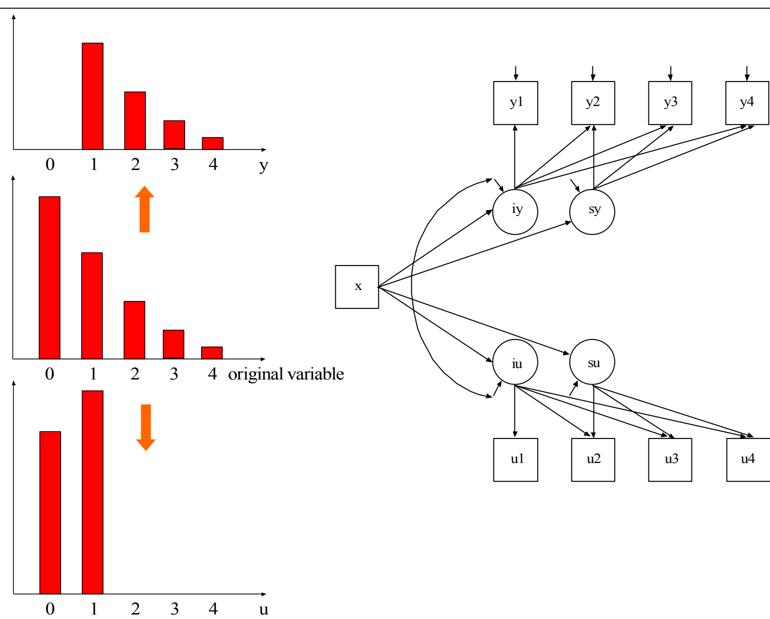
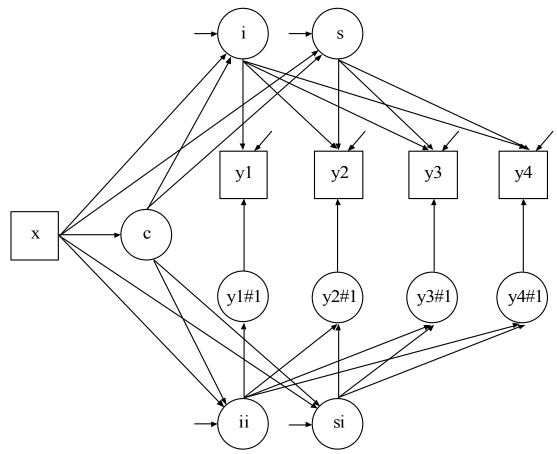
Two-Part (Semicontinuous) Growth Modeling



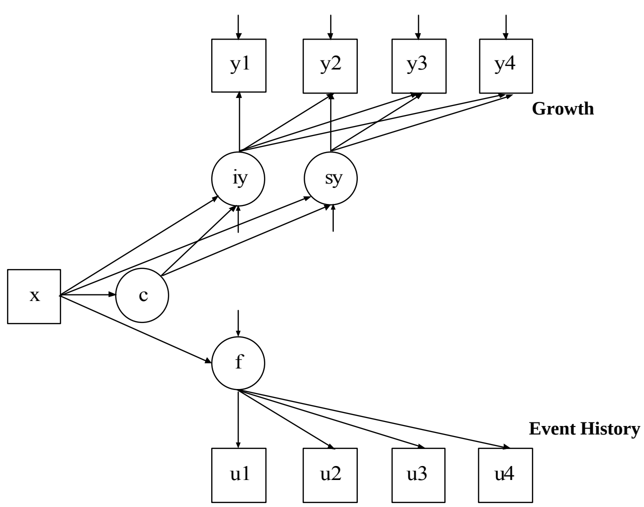
142

Inflated Growth Modeling (Two Classes At Each Time Point)



143

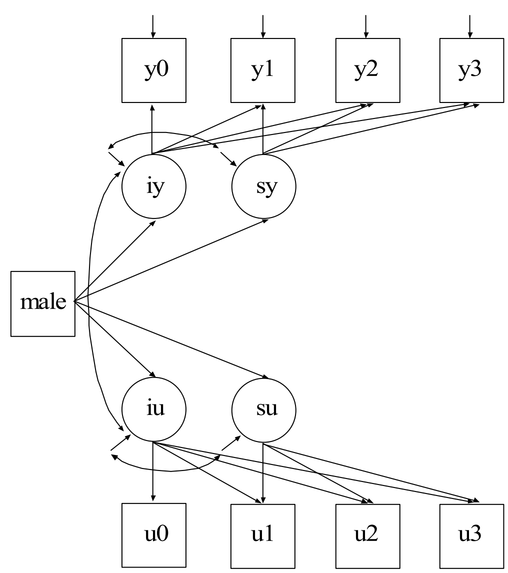
Onset (Survival) Followed By Growth



144

Two-Part Growth Modeling

145



146

Input For Step 1 Of A Two-Part Growth Model

```
TITLE:      step 1 of a two-part growth model
               Amover u    y
                >0    1   >0
                 0    0   999
                999  999  999
DATA:       FILE = amp.dat;
VARIABLE:   NAMES ARE caseid
            amover0 ovrdrnk0 illdrnk0 vrydrn0
            amover1 ovrdrnk1 illdrnk1 vrydrn1
            amover2 ovrdrnk2 illdrnk2 vrydrn2
            amover3 ovrdrnk3 illdrnk3 vrydrn3
            amover4 ovrdrnk4 illdrnk4 vrydrn4
            amover5 ovrdrnk5 illdrnk5 vrydrn5
            amover6 ovrdrnk6 illdrnk6 vrydrn6
            tfq0-tfq6 v2 sex race livewith
            agedrnk0-agedrnk6 grades0-grades6;
            USEV = amover0 amover1 amover2 amover3
            sex race u0-u3 y0-y3;
          ! MISSING = ALL (999);
```

147

Input For Step 1 Of A Two-Part Growth Model (Continued)

```
DEFINE:       u0 = 1;                            !binary part of variable
              IF(amover0 eq 0) THEN u0 = 0;
              IF(amover0 eq 999) THEN u0 = 999;
              y0 = amover0;                      !continuous part of variable
              IF (amover0 eq 0) THEN y0 = 999;
              u1 = 1;
              IF(amover1 eq 0) THEN u1 = 0;
              IF(amover1 eq 999) THEN u1 = 999:
              y1 = amover1;
              IF(amover1 eq 0) THEN y1 = 999;
              u2 = 1;
              IF(amover2 eq 0) THEN u2 = 0;
              IF(amover2 eq 999) THEN u2 = 999;
              y2 = amover2;
              IF(amover2 eq 0) THEN y2 = 999;
              u3 = 1;
              IF(amover3 eq 0) THEN u3 = 0;
              IF(amover3 eq 999) THEN u3 = 999;
              y3 = amover3;
              IF(amover3 eq 0) THEN y3 = 999;
ANALYSIS:     TYPE = BASIC;
SAVEDATA:     FILE = ampyu.dat;
```

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Output Excerpts Step 1 Of A Two-Part Growth Model

SAVEDATA Information

```
Order and format of variables
   AMOVER0   F10.3
   AMOVER1   F10.3
   AMOVER2   F10.3
   AMOVER3   F10.3
   SEX       F10.3
   RACE      F10.3
   U0        F10.3
   U1        F10.3
   U2        F10.3
   U3        F10.3
   Y0        F10.3
   Y1        F10.3
   Y2        F10.3
   Y3        F10.3
Save file
     ampyu.dat
Save file format
     14F10.3
Save file record length    1000
```


Input For Step 2 Of A Two-Part Growth Model

```
TITLE:     two-part growth model with linear growth for both
           parts
DATA:      FILE = ampyau.dat;
VARIABLE:  NAMES = amover0-amover3 sex race u0-u3 y0-y3;
           USEV = u0-u3 y0-y3 male;
           USEOBS = u0 NE 999;
           MISSING = ALL (999);
           CATEGORICAL = u0-u3;
DEFINE:    male = 2-sex;
```

Input For Step 2 Of A Two-Part Growth Model (Continued)

```
ANALYSIS:   TYPE = MISSING;
            ESTIMATOR = ML;
            ALGORITHM = INTEGRATION;
            COVERAGE = .09;
MODEL:      iu su | u0@0 u1@0.5 u2@1.5 u3@2.5;
            iy sy | y0@0 y1@0.5 y2@1.5 y3@2.5;
            iu-sy ON male;
            ! estimate the residual covariances
            ! iu with su, iy with sy, and iu with iy
            iu WITH sy@0;
            su WITH iy-sy@0;
OUTPUT:     PATTERNS SAMPSTAT STANDARDIZED TECH1 TECH4 TECH8;
PLOT:       TYPE = PLOT3;
            SERIES = u0-u3(su) | y0-y3(sy);
```

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Output Excerpts Step 2 Of A Two-Part Growth Model

Tests of Model Fit

```
Loglikelihood

          H0 Value                    -3277.101

Information Criteria

          Number of Free parameters          19
          Akaike (AIC)                 6592.202
          Bayesian (BIC)               6689.444
          Sample-Size Adjusted BIC     6629.092
             (n* = (n + 2) / 24)
```

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Output Excerpts Step 2 Of A Two-Part Growth Model (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
IU |					
U0	1.000	0.000	0.000	2.839	0.843
U1	1.000	0.000	0.000	2.839	0.882
U2	1.000	0.000	0.000	2.839	0.926
U3	1.000	0.000	0.000	2.839	0.905
SU |					
U0	0.000	0.000	0.000	0.000	0.000
U1	0.500	0.000	0.000	0.416	0.129
U2	1.500	0.000	0.000	1.249	0.407
U3	2.500	0.000	0.000	2.082	0.664

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Output Excerpts Step 2 Of A Two-Part Growth Model (Continued)

IY |					
Y0	1.000	0.000	0.000	0.534	0.787
Y1	1.000	0.000	0.000	0.534	0.738
Y2	1.000	0.000	0.000	0.534	0.740
Y3	1.000	0.000	0.000	0.534	0.644
SY |					
Y0	0.000	0.000	0.000	0.000	0.000
Y1	0.500	0.000	0.000	0.117	0.162
Y2	1.500	0.000	0.000	0.351	0.487
Y3	2.500	0.000	0.000	0.586	0.707

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Output Excerpts Step 2 Of A Two-Part Growth Model (Continued)

```
IU       ON
   MALE            0.569    0.234      2.433    0.200    0.100
SU       ON
   MALE           -0.181    0.119     -1.518   -0.218   -0.109
IY       ON
   MALE            0.149    0.061      2.456    0.279    0.139
SY       ON
   MALE           -0.068    0.038     -1.790   -0.290   -0.145

IU       WITH
   SU             -1.144    0.326     -3.509   -0.484   -0.484
   IY              1.193    0.134      8.897    0.788    0.788
   SY              0.000    0.000      0.000    0.000    0.000
IY       WITH
   SY             -0.039    0.019     -2.109   -0.316   -0.316
SU       WITH
   IY              0.000    0.000      0.000    0.000    0.000
   SY              0.000    0.000      0.000    0.000    0.000
```

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Output Excerpts Step 2 Of A Two-Part Growth Model (Continued)

```
Intercepts
   Y0                0.000    0.000    0.000    0.000    0.000
   Y1                0.000    0.000    0.000    0.000    0.000
   Y2                0.000    0.000    0.000    0.000    0.000
   Y3                0.000    0.000    0.000    0.000    0.000
   IU                0.000    0.000    0.000    0.000    0.000
   SU                0.855    0.098    8.716    1.027    1.027
   IY                0.232    0.059    3.901    0.435    0.435
   SY                0.240    0.031    7.830    1.025    1.025

Thresholds
   U0$1              2.655    0.206   12.877
   U1$1              2.655    0.206   12.877
   U2$1              2.655    0.206   12.877
   U3$1              2.655    0.206   12.877
```

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Output Excerpts Step 2 Of A Two-Part Growth Model (Continued)

Residual Variances

Y0	0.175	0.032	5.470	0.175	0.380
Y1	0.266	0.029	9.159	0.266	0.509
Y2	0.238	0.027	8.810	0.238	0.457
Y3	0.269	0.054	5.014	0.269	0.392
IU	7.982	1.086	7.351	0.990	0.990
SU	0.685	0.202	3.400	0.988	0.988
IY	0.279	0.040	7.019	0.981	0.981
SY	0.054	0.017	3.224	0.979	0.979

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Output Excerpts Step 2 Of A Two-Part Growth Model (Continued)

Observed Variable	R-Square
U0	0.710
U1	0.682
U2	0.650
U3	0.666
Y0	0.620
Y1	0.491
Y2	0.543
Y3	0.608

Latent Variable	R-Square
IU	0.010
SU	0.012
IY	0.019
SY	0.021

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Output Excerpts Step 2 Of A Two-Part Growth Model (Continued)

Technical 4 Output

ESTIMATED MEANS FOR THE LATENT VARIABLES

	IU	SU	IY	SY	MALE
1	0.305	0.758	0.312	0.204	0.536

ESTIMATED COVARIANCE MATRIX FOR THE LATENT VARIABLES

	IU	SU	IY	SY	MALE
IU	8.062				
SU	-1.170	0.694			
IY	1.214	-0.007	0.285		
SY	-0.010	0.003	-0.042	0.055	
MALE	0.142	-0.045	0.037	-0.017	0.249

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Output Excerpts Step 2 Of A Two-Part Growth Model (Continued)

ESTIMATED CORRELATION MATRIX FOR THE LATENT VARIABLES

	IU	SU	IY	SY	MALE
IU	1.000				
SU	-0.495	1.000			
IY	0.801	-0.015	1.000		
SY	-0.014	0.016	-0.336	1.000	
MALE	0.100	-0.109	0.139	-0.145	1.000

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Multiple Populations

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Multiple Population Growth Modeling

- Group as a dummy variable
- Multiple-group analysis
- Multiple-group analysis of randomized interventions

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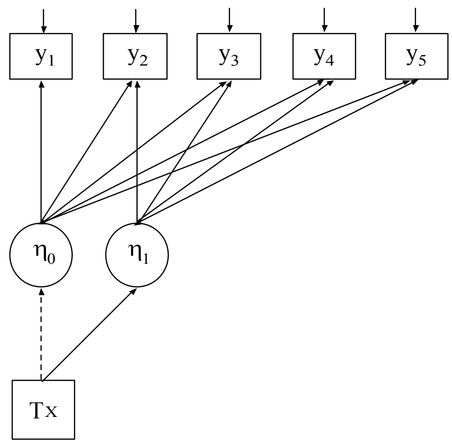
Group Dummy Variable As A Covariate



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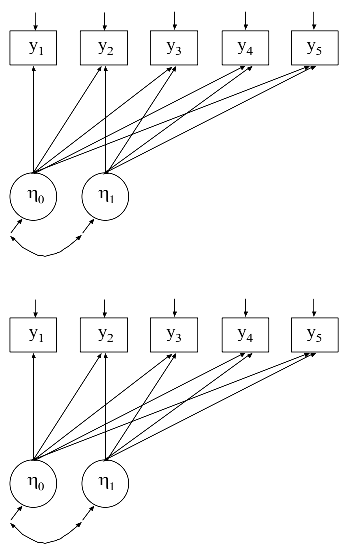
Two-Group Model



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Multiple Population Growth Modeling Specifications

Let y_{git} denote the outcome for population (group) g, individual i, and timepoint t,

$$\text{Level 1: } y_{gti} = \eta_{g0i} + \eta_{g1i}\, x_t + \varepsilon_{gti}, \tag{65}$$

$$\text{Level 2a: } \eta_{g0i} = \alpha_{g0} + \gamma_{g0}\, w_{gi} + \zeta_{g0i}, \tag{66}$$

$$\text{Level 2b: } \eta_{g1i} = \alpha_{g1} + \gamma_{g1}\, w_{gi} + \zeta_{g1i}, \tag{67}$$

Measurement invariance (level-1 equation): time-invariant intercept 0 and slopes 1, x_t

Structural differences (level-2): α_g , γ_g , $V(\zeta_g)$

Alternative parameterization:

$$\text{Level 1: } y_{gti} = \nu + \eta_{g0i} + \eta_{g1i}\, x_t + \varepsilon_{gti}, \tag{68}$$

with α_{10} fixed at zero in level 2a.

Analysis steps:

1. Separate growth analysis for each group
2. Joint analysis of all groups, free structural parameters
3. Join analysis of all groups, tests of structural parameter invariance

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NLSY: Multiple Cohort Structure

Birth Year Cohort	Age[a] 18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
57								82	83	84	85	86	87	88	89	90	91	92	93	94
58							82	83	84	85	86	87	88	89	90	91	92	93	94	
59						82	83	84	85	86	87	88	89	90	91	92	93	94		
60					82	83	84	85	86	87	88	89	90	91	92	93	94			
61				82	83	84	85	86	87	88	89	90	91	92	93	94				
62			82	83	84	85	86	87	88	89	90	91	92	93	94					
63		82	83	84	85	86	87	88	89	90	91	92	93	94						
64	82	83	84	85	86	87	88	89	90	91	92	93	94							

[a] Non-shaded areas represent years in which alcohol measures were obtained

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Multiple Group Modeling Of Multiple Cohorts

- Data – two cohorts born in 1961 and 1962 measured on the frequency of heavy drinking in the years 1983, 1984, 1988, and 1989
- Development of heavy drinking across chronological age, not year of measurement, is of interest

Cohort/Year	1983	1984	1988	1989
1961 (older)	**22**	23	**27**	28
1962 (younger)	21	**22**	26	**27**

Cohort/Age	21	22	23	24	25	26	27	28
1961 (older)		**83**	**84**				**88**	**89**
1962 (younger)	**83**	**84**				**88**	**89**	

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Multiple Group Modeling Of Multiple Cohorts (Continued)

- Time scores calculated for age, not year of measurement

Age	**21**	**22**	**23**	24	25	**26**	**27**	**28**
Time score	**0**	**1**	**2**	3	4	**5**	**6**	**7**

Cohort 1961 time scores 1 2 6 7
Cohort 1962 time scores 0 1 5 6

- Can test the degree of measurement and structural invariance
  - Test of full invariance
    - Growth factor means, variances, and covariances held equal across cohorts
    - Residual variances of shared ages held equal across cohorts

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Input For Multiple Group Modeling Of Multiple Cohorts

```
TITLE:        Multiple Group Modeling Of Multiple Cohorts
DATA:         FILE IS cohort.dat;
VARIABLE:     NAMES ARE cohort hd83 hd84 hd88 hd89;
              MISSING ARE *;
              USEV = hd83 hd84 hd88 hd89;
              GROUPING IS cohort (61 = older 62 = younger);
MODEL:        i s | hd83@0 hd84@1 hd88@5 hd89@6;
              [i] (1);
              [s] (2);
              i (3);
              s (4);
              i WITH s (5);
```

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Input For Multiple Group Modeling Of Multiple Cohorts (Continued)

```
MODEL older:
              i s | hd83@1 hd84@2 hd88@6 hd89@7;
              hd83 (6);
              hd88 (7);
MODEL younger:
              hd84 (6);
              hd89 (7);
OUTPUT:       STANDARDIZED;
```

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Output Excerpts Multiple Group Modeling Of Multiple Cohorts

Tests Of Model Fit

```
Chi-Square Test of Model Fit
          Value                         68.096
          Degrees of Freedom                17
          P-Value                        .0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                        .047
          90 Percent C.I.                 .036   .059
```


Output Excerpts Multiple Group Modeling Of Multiple Cohorts (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
Group OLDER					
I WITH					
S	-.111	.010	-11.390	-.537	-.537
Residual Variances					
HD83	1.141	.046	24.996	1.141	.445
HD84	1.062	.057	18.489	1.062	.453
HD88	1.028	.041	25.326	1.028	.455
HD89	.753	.053	14.107	.753	.358
Variances					
I	1.618	.068	23.651	1.000	1.000
S	.026	.002	13.372	1.000	1.000
Means					
I	1.054	.030	35.393	.828	.828
S	-.032	.005	-6.611	-.200	-.200

Output Excerpts Multiple Group Modeling Of Multiple Cohorts (Continued)

GROUP YOUNGER

Residual Variances

HD83	1.049	.066	15.916	1.049	.393
HD84	1.141	.046	24.996	1.141	.445
HD88	1.126	.056	19.924	1.126	.491
HD89	1.028	.041	25.326	1.028	.455

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Preventive Interventions Randomized Trials

Prevention Science Methodology Group (PSMG)

Developmental Epidemiological Framework:

- Determining the levels and variation in risk and protective factors as well as developmental paths within a defined population in the absence of intervention
- Directing interventions at these risk and protective factors in an effort to change the developmental trajectories in a defined population
- Evaluating variation in intervention impact across risk levels and contexts on proximal and distal outcomes, thereby empirically testing the developmental model

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Aggressive Classroom Behavior: The GBG Intervention

Muthén & Curran (1997, Psychological Methods)

The Johns Hopkins Prevention Center carried out a school-based preventive intervention randomized trial in Baltimore public schools starting in grade 1. One of the interventions tested was the Good Behavior Game intervention, a classroom based behavior management strategy promoting good behavior. It was designed specifically to reduce aggressive behavior of first graders and was aimed at longer term impact on aggression through middle school.

One first grade classroom in a school was randomly assigned to receive the Good Behavior Game intervention and another matched classroom in the school was treated as control. After an initial assessment in fall of first grade, the intervention was administered during the first two grades.

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Aggressive Classroom Behavior: The GBG Intervention (Continued)

The outcome variable of interest was teacher ratings (TOCA-R) of each child's aggressive behavior (breaks rules, harms property, fights, etc.) in the classroom through grades 1 – 6. Eight teacher ratings were made from fall and spring for the first two grades and every spring in grades 3 – 6.

The most important scientific question was whether the Good Behavior Game reduces the slope of the aggression trajectory across time. It was also of interest to know whether the intervention varies in impact for children who started out as high aggressive versus low aggressive.

Analyses in Muthén-Curran (1997) were based on data for 75 boys in the GBG group who stayed in the intervention condition for two years and 111 boys in the control group.

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The GBG Aggression Example: Analysis Results

Muthén & Curran (1997):

- Step 1: Control group analysis
- Step 2: Treatment group analysis
- Step 3: Two-group analysis w/out interactions
- Step 4: Two-group analysis with interactions
- Step 5: Sensitivity analysis of final model
- Step 6: Power analysis

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MUTHÉN AND CURRAN

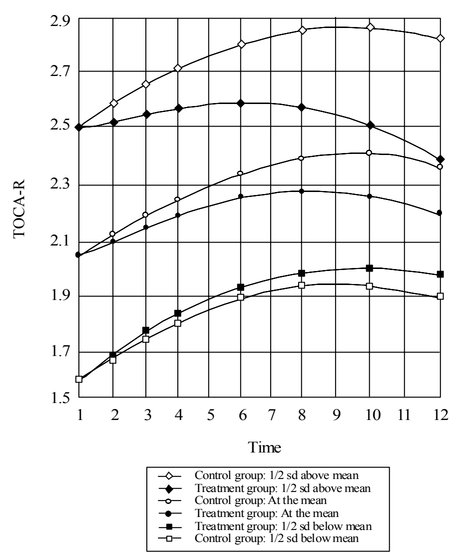

Figure 15. Model implied growth trajectories of Teacher Observation of Classroom Behavior—Revised (TOCA-R) scores as a function of initial status. Each timepoint represents one 6-month interval.

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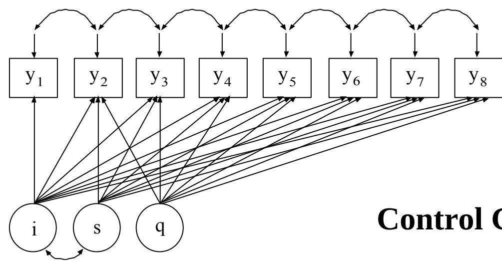

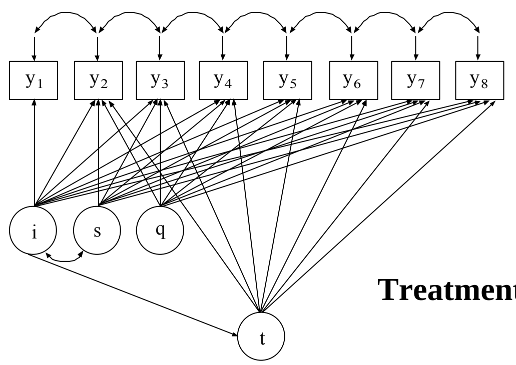

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Input Excerpts For Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects

```
TITLE:          Aggressive behavior intervention growth model
                n = 111 for control group
                n = 75 for tx group
MODEL:          i s q | y1@0 y2@1 y3@2 y4@3 y5@5 y6@7 y7@9 y8@11;
                i t | y1@0 y2@1 y3@2 y4@3 y5@5 y6@7 y7@9 y8@11;
                [y1-y8] (1);   !alternative growth model
                [i@0];         !parameterization
                i (2);
                s (3);
                i WITH s (4);
                [s] (5);
                [q] (6);
                t@0 q@0;
                q WITH i@0 s@0 t@0; y1-y7 PWITH y2-y8;
                t ON i;
```

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Input Excerpts For Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

```
MODEL control:
                [s] (5);
                [q] (6);
                t ON i@0;
                [t@0];
```

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Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects

Tests Of Model Fit

```
Chi-Square Test of Model Fit

          Value                        64.553
          Degrees of Freedom               50
          P-Value                       .0809

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                       .056
          90 Percent C.I.                .000   .092
```

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Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

Group Control

Observed Variable	R-Square
Y1	.644
Y2	.642
Y3	.663
Y4	.615
Y5	.637
Y6	.703
Y7	.812
Y8	.818

Group Tx

Observed Variable	R-Square
Y1	.600
Y2	.623
Y3	.568
Y4	.464
Y5	.425
Y6	.399
Y7	.703
Y8	.527

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Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

Group Control	Estimates	S.E.	Est./S.E.	Std	StdYX
T ON					
I	**.000**	**.000**	**.000**	**999.000**	**999.000**
Residual Variances					
Y1	.444	.088	5.056	.444	.356
Y2	.449	.079	5.714	.449	.358
Y3	.414	.069	6.026	.414	.337
Y4	.522	.080	6.551	.522	.385
Y5	.512	.079	6.469	.512	.363
Y6	.422	.074	5.677	.422	.297
Y7	.264	.083	3.186	.264	.188
Y8	.291	.094	3.097	.291	.182
T	.000	.000	.000	999.000	999.000
Variances					
I	.803	.109	7.330	1.000	1.000
S	.004	.001	3.869	1.000	1.000
Q	.000	.000	.000	999.000	999.000

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Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Means					
I	.000	.000	.000	.000	.000
S	.086	.021	4.035	1.285	1.285
Q	-.005	.002	-3.005	999.000	999.000
Intercepts					
Y1	2.041	.078	26.020	2.041	1.828
Y2	2.041	.078	26.020	2.041	1.823
Y3	2.041	.078	26.020	2.041	1.841
Y4	2.041	.078	26.020	2.041	1.753
Y5	2.041	.078	26.020	2.041	1.718
Y6	2.041	.078	26.020	2.041	1.711
Y7	2.041	.078	26.020	2.041	1.724
Y8	2.041	.078	26.020	2.041	1.612
T	.000	.000	.000	999.000	999.000

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Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

Group Tx

	Estimates	S.E.	Est./S.E.	Std	StdYX
T ON					
I	**-.052**	**.015**	**-3.347**	**-1.000**	**-1.000**
Residual Variances					
Y1	.535	.141	3.801	.535	.400
Y2	.439	.122	3.595	.439	.377
Y3	.501	.108	4.653	.501	.432
Y4	.701	.132	5.332	.701	.536
Y5	.736	.133	5.545	.736	.575
Y6	.805	.152	5.288	.805	.601
Y7	.245	.104	2.364	.245	.297
Y8	.609	.182	3.351	.609	.473
T	.000	.000	.000	.000	.000
Variances					
I	.803	.109	7.330	1.000	1.000
S	.004	.001	3.869	1.000	1.000
Q	.000	.000	.000	999.000	999.000

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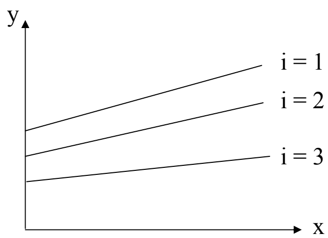
Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Means					
I	.000	.000	.000	.000	.000
S	.086	.021	4.035	1.285	1.285
Q	-.005	.002	-3.005	999.000	999.000
Intercepts					
Y1	2.041	.078	26.020	2.041	1.764
Y2	2.041	.078	26.020	2.041	1.893
Y3	2.041	.078	26.020	2.041	1.895
Y4	2.041	.078	26.020	2.041	1.785
Y5	2.041	.078	26.020	2.041	1.805
Y6	2.041	.078	26.020	2.041	1.764
Y7	2.041	.078	26.020	2.041	2.248
Y8	2.041	.078	26.020	2.041	1.799
T	-.016	.013	-1.225	-.341	-.341

Growth Mixture Modeling

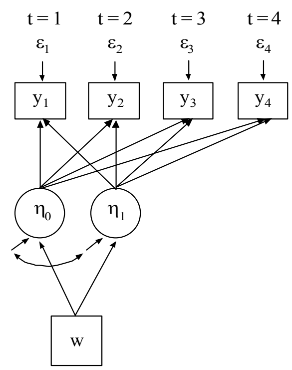
Individual Development Over Time



(1) $y_{ti} = \eta_{0i} + \eta_{1i} x_t + \varepsilon_{ti}$

(2a) $\eta_{0i} = \alpha_0 + \gamma_0 w_i + \zeta_{0i}$

(2b) $\eta_{1i} = \alpha_1 + \gamma_1 w_i + \zeta_{1i}$



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Mixtures And Latent Trajectory Classes

Modeling motivated by substantive theories of:

- Multiple Disease Processes: Prostate cancer (Pearson et al.)
- Multiple Pathways of Development: Adolescent-limited versus life-course persistent antisocial behavior (Moffitt), crime curves (Nagin), alcohol development (Zucker, Schulenberg)
- Subtypes: Subtypes of alcoholism (Cloninger, Zucker)

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Example: Mixed-Effects Regression Models For Studying The Natural History Of Prostate Disease

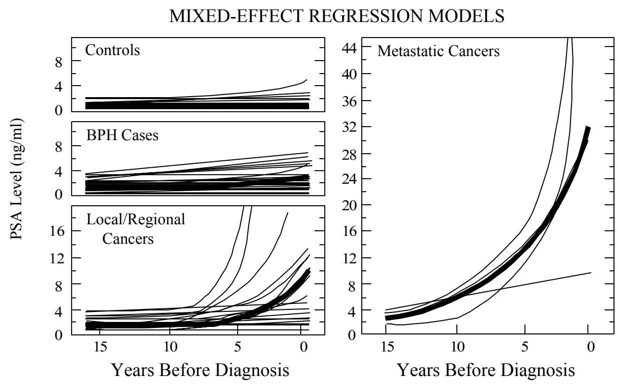

Figure 2. Longitudinal PSA curves estimated from the linear mixed-effects model for the group average (thick solid line) and for each individual in the study (thin solid lines)

Source: Pearson, Morrell, Landis and Carter (1994), Statistics in Medicine

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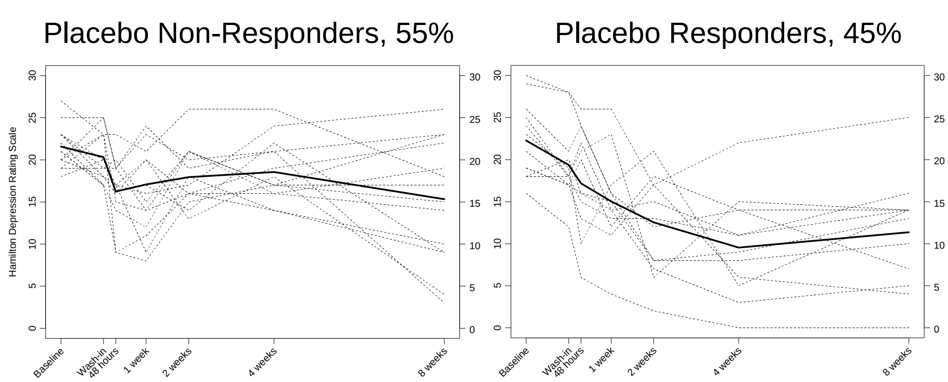
A Clinical Trial Of Depression Medication: Two-Class Growth Mixture Modeling



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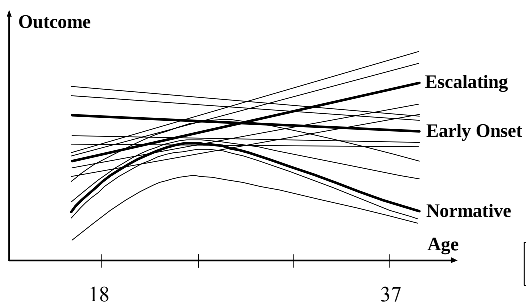
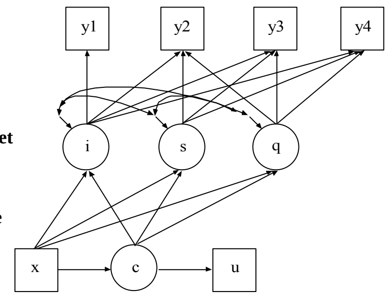
Growth Mixture Modeling
Of Developmental Pathways





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General Growth Mixture Modeling

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General Growth Mixture Modeling (GGMM)

GGMM goes beyond conventional random effect growth modeling by using latent trajectory classes which

- Allow for heterogeneity with respect to
  - Growth functions – different classes correspond to different growth shapes
  - Antecedents – different background variables have different importance for different classes
  - **Consequences – class membership predicts later outcomes**
- Allow for prediction of trajectory class membership
- Allow for confirmatory clustering
  - With respect to parameters – describing curve shapes
  - With respect to typical individuals – known classes
- Allow for classification of individuals
  - Early prediction of problematic development
- Allow for enhanced preventive intervention analysis
  - Different classes benefit differently and can receive different treatments

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General Growth Mixture Modeling (Continued)

- Applications
  - LSAY math achievement development and high school dropout
  - The development of heavy drinking ages 18-30 (NLSY) related to antecedents and consequences. National sample, n = 922

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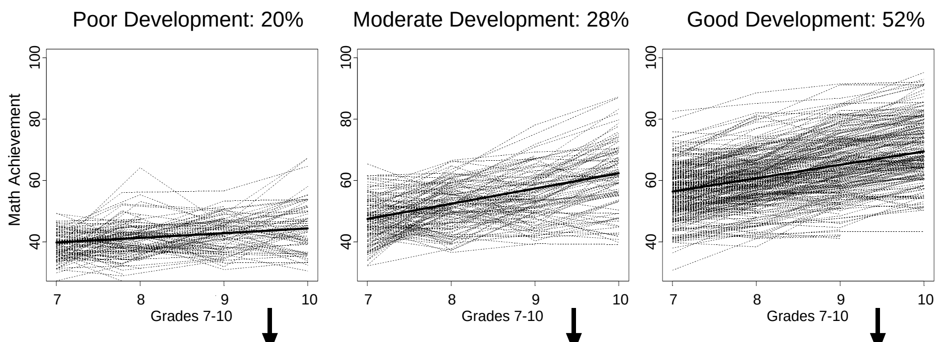
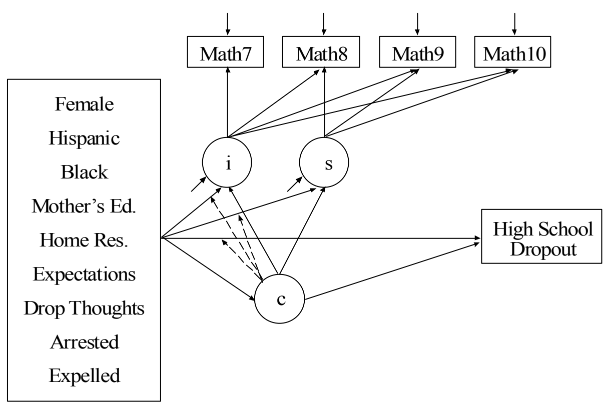
Mplus Graphics For LSAY Math Achievement Trajectory Classes

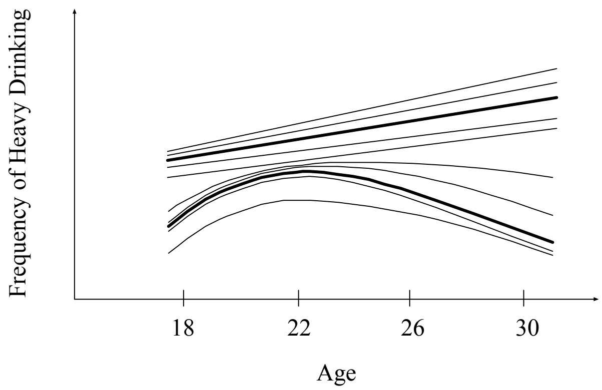


Dropout:	69%	8%	1%

LSAY Math Achievement Trajectory Classes



Example: NLSY Heavy Drinking
Two Latent Trajectory Classes



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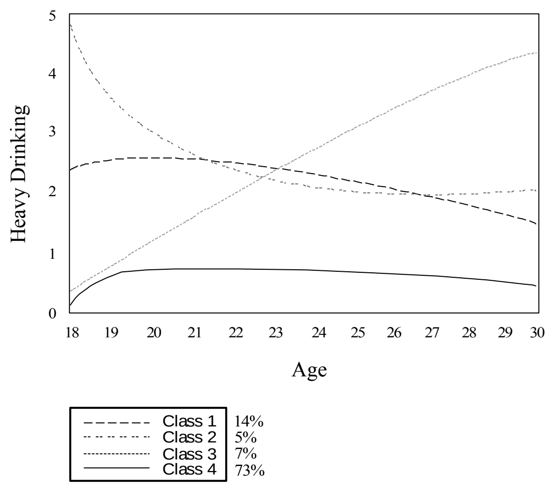
NLSY: Heavy Drinking, Cohort 64



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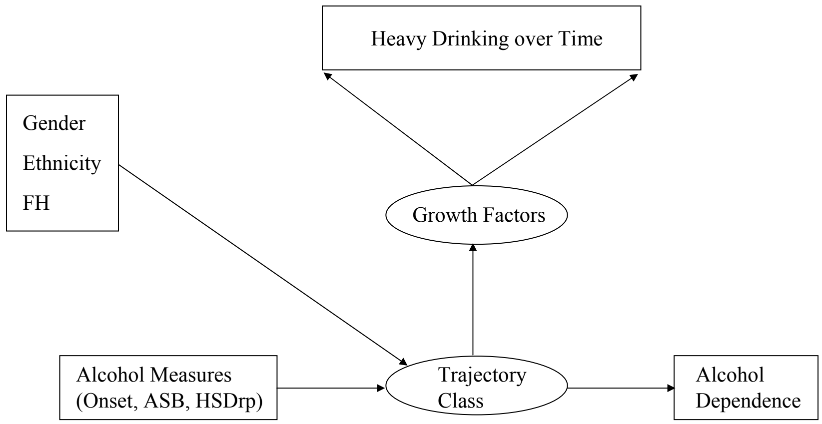
NLSY: Antecedents And Consequences



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Multinomial Logistic Regression Of c ON x

The multinomial logistic regression model expresses the probability that individual i falls in class k of the latent class variable c as a function of the covariate x,

$$P(c_j = k \mid x_i) = \frac{e^{\alpha_k + \gamma_k x_i}}{\sum_{s=1}^{K} e^{\alpha_s + \gamma_s x_i}}, \qquad (90)$$

where $\alpha_\kappa = 0$, $\gamma_\kappa = 0$ so that $e^{\alpha_K + \gamma_K x_i} = 1$.

This implies that the log odds comparing class k to the last class K is

$$log[P(c_i = k \mid x_i)/P(c_i = K \mid x_i)] = \alpha_k + \gamma_k x_i. \qquad (91)$$

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Heavy Drinking And Alcohol Dependence NLSY Cohort64 (n=922)

HD Class on Covariates						
	HD Classes					
	1 (Down)		2 (High18)		3(Up)	
	Est.	t	Est.	t	Est.	t
Male	**1.21**	5.52	**1.25**	3.48	**1.45**	4.73
Black	**-0.89**	-3.43	**-3.14**	-2.86	-0.06	-0.17
Hisp	**-0.65**	-2.22	-0.35	-0.86	-0.01	-0.03
ES	**1.24**	4.79	**2.05**	5.72	0.71	1.78
FH1	0.03	0.09	-0.21	-0.41	-0.08	-0.16
FH23	0.04	0.15	0.25	0.56	0.08	0.23
FH123	-0.23	-0.58	**1.18**	2.59	**1.00**	2.60
HSDRP	0.57	1.98	0.32	0.76	**0.91**	2.93
Coll	-0.07	-0.31	**-1.31**	-2.85	**-1.08**	-2.59

Heavy Drinking And Alcohol Dependence NLSY Cohort64 (n=922) (Continued)

Alcohol Dependence as a Function of Heavy Drinking Class

	Threshold	Probability	Odds	Odds Ratio
HD Class 1 (Down)	1.631	0.164	0.196	3.92
HD Class 2 (High 18)	1.041	0.261	0.353	7.06
HD Class 3 (Up)	-0.406	0.600	1.500	30.00
HD Class 4 (Norm)	2.987	0.048	0.050	1.00

Probability = $1/(1 + e^{-Logit})$ where Logit = − threshold

Odds = Probability/(1 - Probability)

Odds Ratio = Odds (class k)/Odds (class K) for K = 4

Input Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome

```
TITLE:      NLSY for cohort 64 quadratic growth mixture model with
            covariates centered at 25: four-class model of heavy
            drinking with classes predicting dep94

VARIABLE:   CLASSES = c(4);
            CATEGORICAL IS dep94;  ! dep94 is alcohol dependence

ANALYSIS:   TYPE = MIXTURE;

MODEL:      %OVERALL%
            c#1-c#3 ON male black hisp es fh1 fh23 fh123 hsdrp coll;
            i s1 s2 | hd82@-3.008 hd83@-2.197 hd84@-1.621 hd88@-.235
                      hd89@.000 hd94@.884;
```

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Input Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome (Continued)

```
!       log age scale: x_t = a*(ln(t-b) – ln(c-b));
!       where t is time, a and b are constants to fit the mean curve
!       (chosen as a = 2 and b = 16), and c is the centering age,
!       here set at 25.
      %c#1%                              ! Not needed
      [dep94$1*1 i*2 s1*-.5 s2*-.1];     ! Not needed
      %c#2%                              ! Not needed
      [dep94$1*0 i*1 s1*-.2 s2*-.3];     ! Not needed
      %c#3%                              ! Not needed
      [dep94$1*.6 i*3 s1*1.5 s2*.2];     ! Not needed
      %c#4%                              ! Not needed
      [dep94$1*2 i*.6 s1*-.2 s2*-.1];    ! Not needed
```

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Output Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome

C#1 ON			
MALE	1.214	.220	5.515
BLACK	-.886	.258	-3.434
HISP	-.645	.290	-2.223
ES	1.240	.259	4.789
FH1	.026	.291	.088
FH23	.039	.261	.149
FH123	-.233	.399	-.583
HSDRP	.566	.286	1.976
COLL	-.071	.231	-.308

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Output Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome (Continued)

C#2 ON			
MALE	1.248	.359	3.481
BLACK	-3.138	1.097	-2.860
HISP	-.346	.401	-.864
ES	2.045	.357	5.722
FH1	-.211	.514	-.410
FH23	.247	.444	.556
FH123	1.178	.456	2.585
HSDRP	.323	.428	.756
COLL	-1.311	.460	-2.851
C#3 ON			
MALE	1.454	.308	4.727
BLACK	-.059	.344	-.171
HISP	-.011	.369	-.030
ES	.712	.399	1.784
FH1	-.079	.502	-.157
FH23	.084	.364	.232
FH123	1.004	.387	2.596
HSDRP	.913	.312	2.926
COLL	-1.075	.414	-2.594

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Output Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome (Continued)

```
Class 1
 Thresholds
  DEP94$1                1.631        0.248        6.574

Class 2
 Thresholds
  DEP94$1                1.041        0.338        3.077

Class 3
 Thresholds
  DEP94$1               -0.406        0.272       -1.493

Class 4
 Thresholds
  DEP94$1                2.987        0.208       14.392
```

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Output Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome (Continued)

Classification Information

```
FINAL CLASS COUNTS AND PROPORTIONS OF TOTAL SAMPLE SIZE
  Class 1         135.95653        0.14746
  Class 2          45.86689        0.04975
  Class 3          74.68767        0.08101
  Class 4         665.48891        0.72179
CLASSIFICATION OF INDIVIDUALS BASED ON THEIR MOST LIKELY CLASS
MEMBERSHIP
Class Counts and Proportions

  Class 1               134        0.14534
  Class 2                46        0.04989
  Class 3                72        0.07809
  Class 4               670        0.72668
Average Latent Class Probabilities for Most Likely Latent Class
Membership (Row) by Latent Class (Column)
                      1        2        3         4
  Class 1         0.994    0.000    0.000     0.005
  Class 2         0.003    0.997    0.000     0.000
  Class 3         0.007    0.000    0.947     0.047
  Class 4         0.003    0.000    0.010     0.987
```

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Growth Mixtures In Randomized Trials

Different treatment effects in different trajectory classes

Muthén, B., Brown, C.H., Masyn, K., Jo, B., Khoo, S.T., Yang, C.C., Wang, C.P. Kellam, S., Carlin, J., & Liao, J. (2002). General growth mixture modeling for randomized preventive interventions. Biostatistics, 3, 459-475.

See also Muthen & Curran, 1997 for monotonic treatment effects

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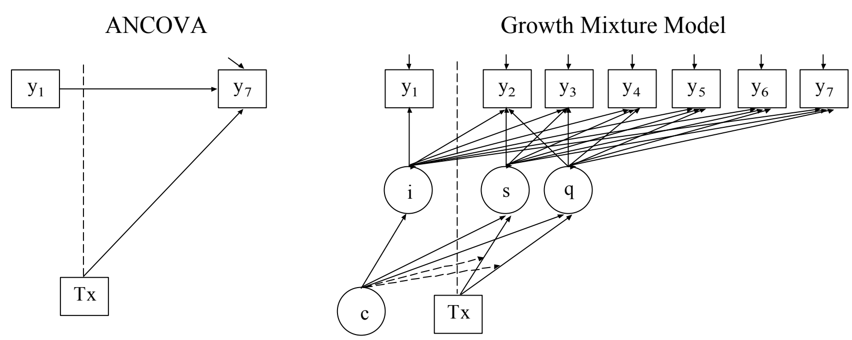
Modeling Treatment Effects



- GMM: treatment changes trajectory shape

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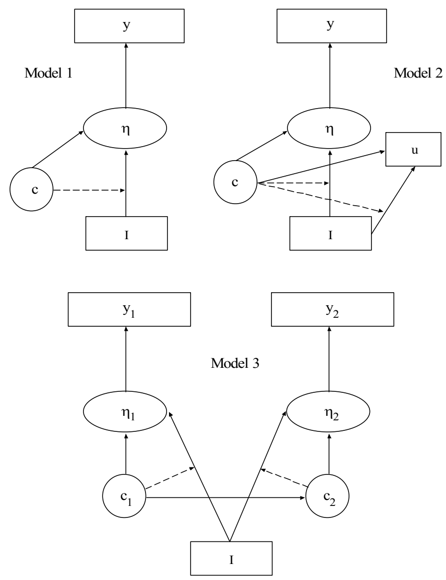

Figure 1. *Path Diagrams for Models 1 - 3*

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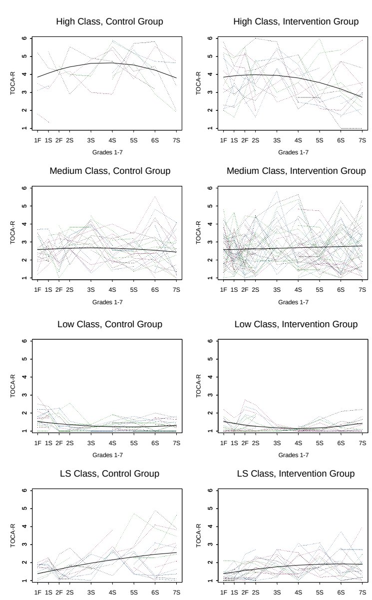

Figure 6. *Estimated Mean Growth Curves and Observed Trajectories for 4-Class model 1 by Class and Intervention Status*

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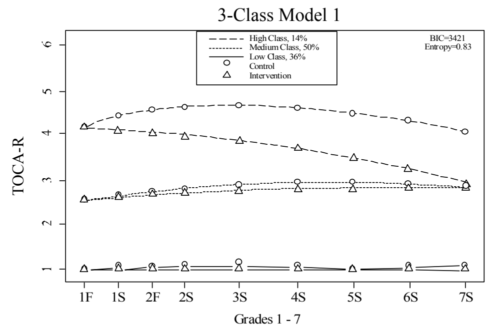

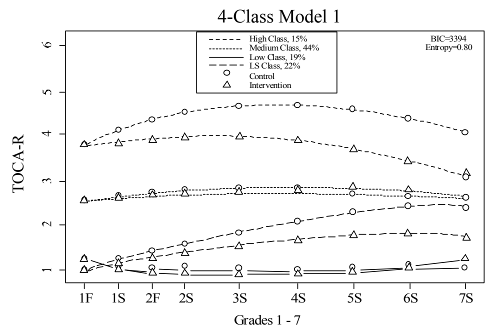

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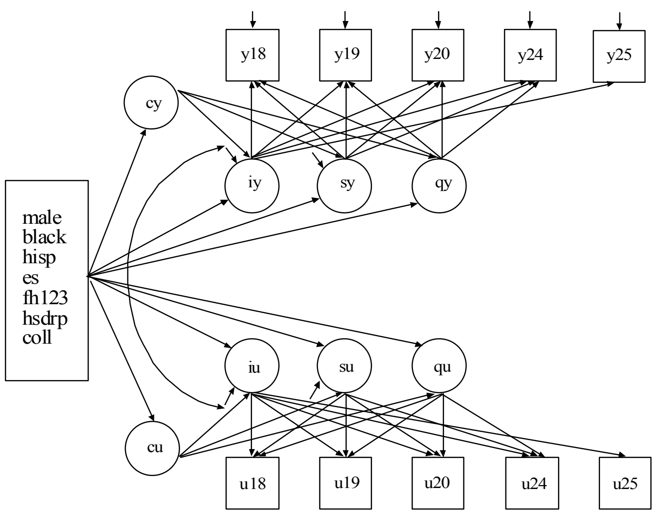
Two-Part Growth Mixture Modeling



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Further Readings On General Growth Mixture Modeling

Muthén, B. (2004). Latent variable analysis: Growth mixture modeling and related techniques for longitudinal data. In D. Kaplan (ed.), Handbook of quantitative methodology for the social sciences (pp. 345-368). Newbury Park, CA: Sage Publications. (#100)

Muthén, B. & Shedden, K. (1999). Finite mixture modeling with mixture outcomes using the EM algorithm. Biometrics, 55, 463-469. (#78)

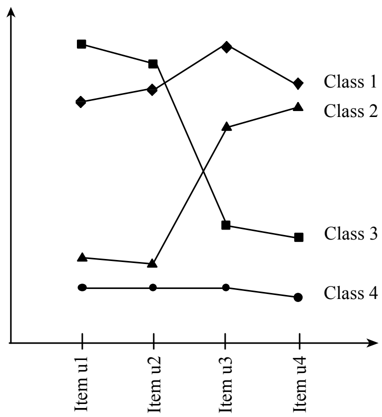
Muthén, B., Brown, C.H., Masyn, K., Jo, B., Khoo, S.T., Yang, C.C., Wang, C.P., Kellam, S., Carlin, J. & Liao, J. (2002). General growth mixture modeling for randomized preventive interventions. Biostatistics, 3, 459-475. (#87)

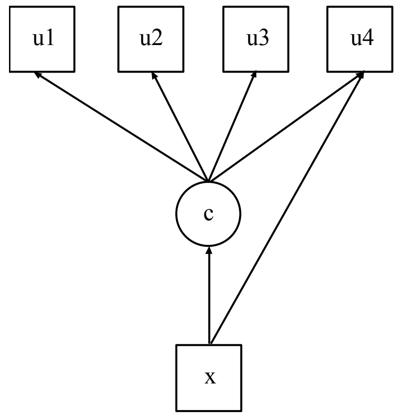
Muthén, B., Khoo, S.T., Francis, D. & Kim Boscardin, C. (2002). Analysis of reading skills development from Kindergarten through first grade: An application of growth mixture modeling to sequential processes. In S.R. Reise & N. Duan (eds), Multilevel modeling: Methodological advances, issues, and applications (pp. 71 – 89). Mahaw, NJ: Lawrence Erlbaum Associates. (#77)

Latent Class Models

Latent Class Analysis

Item Profiles





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Latent Transition Analysis

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Latent Transition Analysis

- Setting
  - Cross-sectional or longitudinal data
  - Multiple items measuring several different constructs
  - Hypothesized simple structure for measurements
  - Hypothesized constructs represented as latent class variables (categorical latent variables)
- Aim
  - Identify items that indicate classes well
  - Test simple measurement structure
  - Study relationships between latent class variables
  - Estimate class probabilities
  - Relate class probabilities to covariates
  - Classify individuals into classes (posterior probabilities)
- Application
  - Latent transition analysis with four latent class indicators at two time points and a covariate

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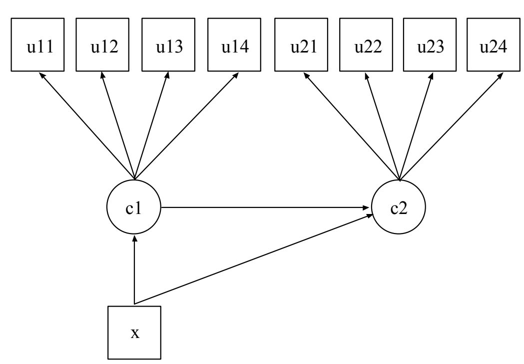
Latent Transition Analysis

Transition Probabilities

		c2 1	c2 2
c1	1	0.8	0.2
c1	2	0.4	0.6

Time Point 1 Time Point 2



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Input For LTA With Two Time Points And A Covariate

```
TITLE:     Latent transition analysis for two time points and a
           covariate

DATA:      FILE = mc2tx.dat;

VARIABLE:  NAMES ARE u11-u14 u21-u24 x xc1 xc2;
           USEV = u11-u14 u21-u24 x;
           CATEGORICAL = u11-u24;
           CLASSES = c1(2) c2(2);

ANALYSIS:  TYPE = MIXTURE;

MODEL:     %OVERALL%
           c2#1 ON c1#1 x;
           c1#1 ON x;
```

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Input For LTA With Two Time Points And A Covariate (Continued)

```
MODEL c1:
%c1#1%
[u11$1-u14$1] (1-4);
%c1#2%
[u11$1-u14$1] (5-8);

MODEL c2:
%c2#1%
[u21$1-u24$1] (1-4);
%c2#2%
[u21$1-u24$1] (5-8);

OUTPUT: TECH1 TECH8;
```

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Output Excerpts LTA With Two Time Points And A Covariate

Tests Of Model Fit

```
Loglikelihood
            H0 Value                       -3926.187
Information Criteria
            Number of Free Parameters             13
            Akaike (AIC)                    7878.374
            Bayesian (BIC)                  7942.175
            Sample-Size Adjusted BIC        7900.886
               (n* = (n + 2) / 24)
            Entropy                            0.902
```

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Output Excerpts LTA With Two Time Points And A Covariate (Continued)

```
Chi-Square Test of Model Fit for the Latent Class Indicator Model
Part
            Pearson Chi-Square

            Value                       250.298
            Degrees of Freedom              244
            P-Value                      0.3772

            Likelihood Ratio Chi-Square

            Value                       240.811
            Degrees of Freedom              244
            P-Value                      0.5457
```

Final Class Counts

```
FINAL  CLASS COUNTS AND PROPORTIONS OF TOTAL SAMPLE SIZE BASED
ON ESTIMATED POSTERIOR PROBABILITIES
 Class 1        328.42644          0.32843
 Class 2        184.43980          0.18444
 Class 3        146.98726          0.14699
 Class 4        340.14650          0.34015
```

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Output Excerpts LTA With Two Time Points And A Covariate (Continued)

Model Results

	Estimates	S.E.	Est./S.E.
Class 1-C1, 1-C2			
Thresholds			
U11$1	-2.020	0.110	-18.353
U12$1	-2.003	0.106	-18.919
U13$1	-1.776	0.098	-18.046
U14$1	-1.861	0.101	-18.396
U21$1	-2.020	0.110	-18.353
U22$1	-2.003	0.106	-18.919
U23$1	-1.776	0.098	-18.046
U24$1	-1.861	0.101	-18.396

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Output Excerpts LTA With Two Time Points And A Covariate (Continued)

Class 1-C1, 2-C2			
Thresholds			
U11$1	-2.020	0.110	-18.353
U12$1	-2.003	0.106	-18.919
U13$1	-1.776	0.098	-18.046
U14$1	-1.861	0.101	-18.396
U21$1	1.964	0.111	17.736
U22$1	2.164	0.119	18.113
U23$1	1.864	0.100	18.704
U24$1	2.107	0.112	18.879
Class 2-C1, 1-C2			
Thresholds			
U11$1	1.964	0.111	17.736
U12$1	2.164	0.119	18.113
U13$1	1.864	0.100	18.704
U14$1	2.107	0.112	18.879
U21$1	-2.020	0.110	-18.353
U22$1	-2.003	0.106	-18.919
U23$1	-1.776	0.098	-18.046
U24$1	-1.861	0.101	-18.396

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Output Excerpts LTA With Two Time Points And A Covariate (Continued)

	Estimates	S.E.	Est./S.E.
Class 2-C1, 2-C2			
Thresholds			
U11$1	1.964	0.111	17.736
U12$1	2.164	0.119	18.113
U13$1	1.864	0.100	18.704
U14$1	2.107	0.112	18.879
U21$1	1.964	0.111	17.736
U22$1	2.164	0.119	18.113
U23$1	1.864	0.100	18.704
U24$1	2.107	0.112	18.879

Output Excerpts LTA With Two Time Points And A Covariate (Continued)

	Estimates	S.E.	Est./S.E.
C2#1 ON			
C1#1	0.530	0.180	2.953
C2#1 ON			
X	-1.038	0.107	-9.703
C1#1 ON			
X	-1.540	0.112	-13.761
Intercepts			
C1#1	0.065	0.082	0.797
C2#1	-0.407	0.120	-3.381

Mover-Stayer Latent Transition Analysis

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Mover-Stayer Latent Transition Analysis

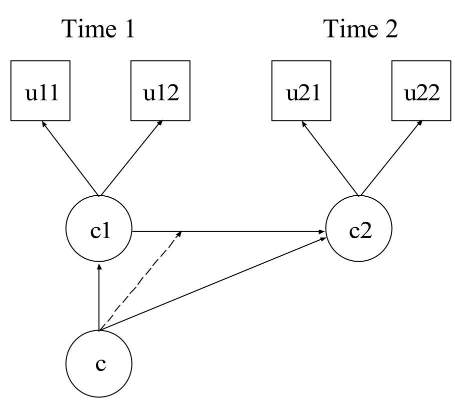
Transition Probabilities

Mover Class (c=1)

c1 \ c2	1	2
1	0.6	0.4
2	0.3	0.7

Stayer Class (c=2)

c1 \ c2	1	2
1	0.90	0.10
2	0.05	0.95



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Multilevel Growth Models

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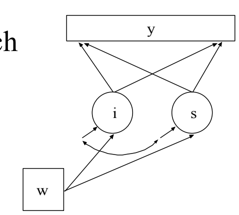
Growth Modeling Approached In Two Ways: Data Arranged As Wide Versus Long

- Wide: Multivariate, Single-Level Approach

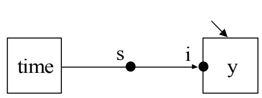


$y_{ti} = i_i + s_i \times \text{time}_{ti} + \varepsilon_{ti}$

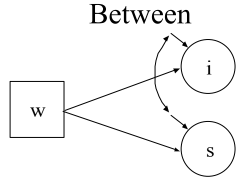
i_i regressed on w_i
s_i regressed on w_i

- Long: Univariate, 2-Level Approach (CLUSTER = id)

Within



Between



The intercept i is called y in Mplus

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Growth Modeling Approached In Two Ways: Data Arranged As Wide Versus Long (Continued)

- Wide (one person):

		t1	t2	t3	t1	t2	t3	
Person i:	id	y1	y2	y3	x1	x2	x3	w

- Long (one cluster):

Person i:	t1	id	y1	x1	w
	t2	id	y2	x2	w
	t3	id	y3	x3	w

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Three-Level Modeling In Multilevel Terms

Time point t, individual i, cluster j.

y_{tij} : individual-level, outcome variable
a_{1tij} : individual-level, time-related variable (age, grade)
a_{2tij} : individual-level, time-varying covariate
x_{ij} : individual-level, time-invariant covariate
w_j : cluster-level covariate

Three-level analysis (Mplus considers Within and Between)

$$\textit{Level 1 (Within)}: \; y_{tij} = \pi_{0ij} + \pi_{1ij}\, a_{1tij} + \pi_{2tij}\, a_{2tij} + e_{tij}, \qquad (1)$$

$$\textit{Level 2 (Within)}: \begin{cases} \pi_{0ij} = \beta_{00j} + \beta_{01j}\, x_{ij} + r_{0ij}, \\ \pi_{1ij} = \beta_{10j} + \beta_{11j}\, x_{ij} + r_{1ij}, \\ \pi_{2tij} = \beta_{20tj} + \beta_{21tj}\, x_{ij} + r_{2tij}. \end{cases} \qquad (2)$$

$$\textit{Level 3 (Between)}: \begin{cases} \beta_{00j} = \gamma_{000} + \gamma_{001}\, w_j + u_{00j}, \\ \beta_{10j} = \gamma_{100} + \gamma_{101}\, w_j + u_{10j}, \\ \beta_{20tj} = \gamma_{200t} + \gamma_{201t}\, w_j + u_{20tj}, \\ \beta_{01j} = \gamma_{010} + \gamma_{011}\, w_j + u_{01j}, \\ \beta_{11j} = \gamma_{110} + \gamma_{111}\, w_j + u_{11j}, \\ \beta_{21tj} = \gamma_{2t0} + \gamma_{2t1}\, w_j + u_{2tj}. \end{cases} \qquad (3)$$

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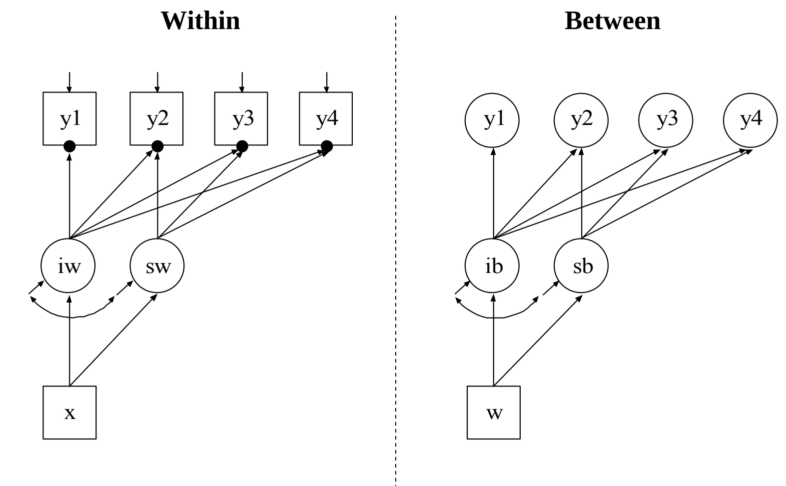
Two-Level Growth Modeling
(Three-Level Modeling)



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Growth Modeling With Multiple Indicators

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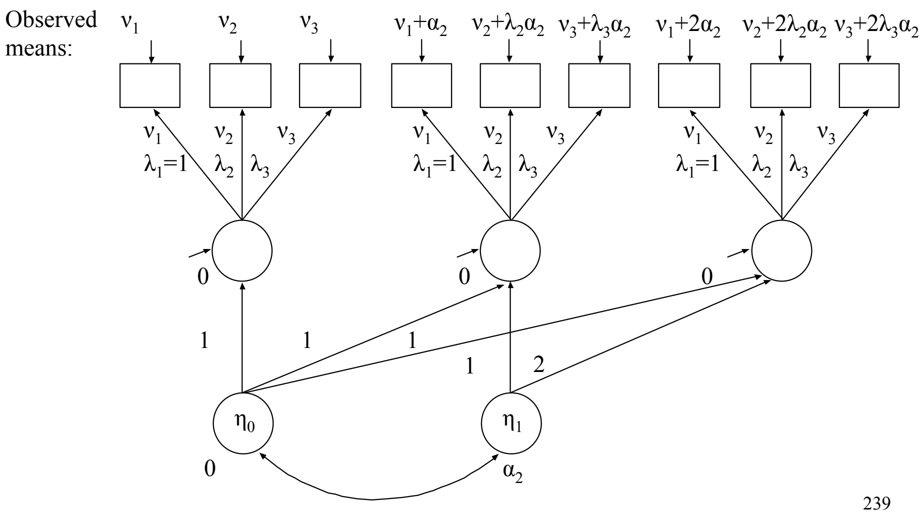
Growth Of Latent Variable Construct Measured By Multiple Indicators



239

Multiple Indicator Growth Modeling Specifications

Let y_{jti} denote the outcome for individual i, indicator j, and timepoint t, and let η_{ti} denote a latent variable construct,

Level 1*a (measurement part):*

$$y_{jti} = \nu_{jt} + \lambda_{jt}\, \eta_{ti} + \varepsilon_{jti}, \qquad (44)$$

Level 1*b* : $\eta_{ti} = \eta_{0i} + \eta_{1i}\; x_t + \zeta_{ti}$, (45)

Level 2*a* : $\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}$, (46)

Level 2*b* : $\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}$, (47)

Measurement invariance: time-invariant indicator intercepts and slopes:

$$\nu_{j1} = \nu_{j2} = \ldots \nu_{jT} = \nu_j, \qquad (48)$$

$$\lambda_{j1} = \lambda_{j2} = \ldots \lambda_{jT} = \lambda_j, \qquad (49)$$

where $\lambda_1 = 1$, $\alpha_0 = 0$. $V(\varepsilon_{jti})$ and $V(\zeta_{ti})$ may vary over time. Structural differences: $E(\eta_{ti})$ and $V(\eta_{ti})$ vary over time.

240

Steps In Growth Modeling With Multiple Indicators

- Exploratory factor analysis of indicators for each timepoint
- Determine the shape of the growth curve for each indicator and the sum of the indicators
- Fit a growth model for each indicator—must be the same
- Confirmatory factor analysis of all timepoints together
  - Covariance structure analysis without measurement parameter invariance
  - Covariance structure analysis with invariant loadings
  - Mean and covariance structure analysis with invariant measurement intercepts and loadings
- Growth model with measurement invariance across timepoints

241

Advantages Of Using Multiple Indicators Instead Of An Average

- Estimation of unequal weights
- Partial measurement invariance—changes across time in individual item functioning
- No confounding of time-specific variance and measurement error variance
- Smaller standard errors for growth factor parameters (more power)

242

Classroom Aggression Data (TOCA)

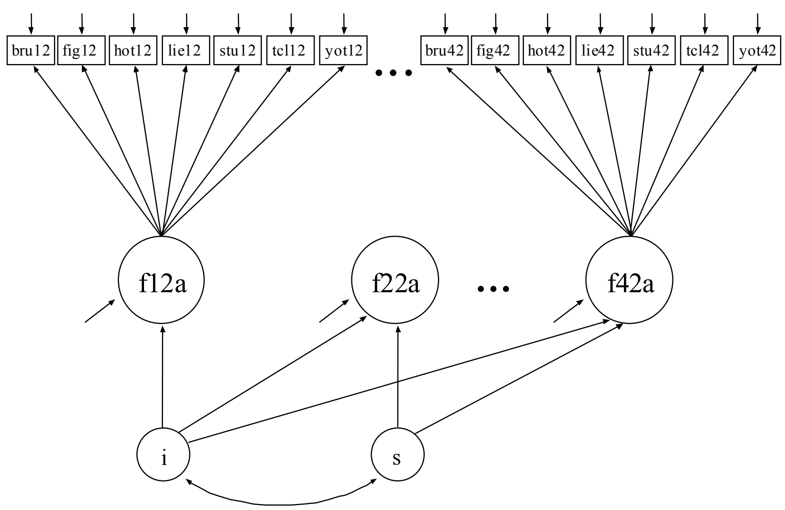
The classroom aggression data are from an intervention study in Baltimore public schools carried out by the Johns Hopkins Prevention Research Center. Subjects were randomized into treatment and control conditions. The TOCA-R instrument was used to measure 10 aggression items at multiple timpoints. The TOCA-R is a teacher rating of student behavior in the classroom. The items are rated on a six-point scale from almost never to almost always.

Data for this analysis include the 342 boys in the control group. Four time points are examined: Spring Grade 1, Spring Grade 2, Spring Grade 3, and Spring Grade 4.

Seven aggression items are used in the analysis:

- Break rules
- Fights
- Harms others
- Lies
- Stubborn
- Teasing classmates
- Yells at others

243



244

Input Excerpts For TOCA Data Multiple Indicator CFA With No Measurement Invariance

```
TITLE:       Multiple indicator CFA with no measurement invariance
.
.
.
MODEL:       f12a BY bru12
                     fig12
                     hot12
                     lie12
                     stu12
                     tcl12
                     yot12;

             f22a BY bru22
                     fig22
                     hot22
                     lie22
                     stu22
                     tcl22
                     yot22;
```

245

Input Excerpts For TOCA Data Multiple Indicator CFA With No Measurement Invariance (Continued)

```
MODEL:       f32a BY bru32
                     fig32
                     hot32
                     lie32
                     stu32
                     tcl32
                     yot32;

             f42a BY bru42
                     fig42
                     hot42
                     lie42
                     stu42
                     tcl42
                     yot42;
```

246

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance

```
TITLE:      Multiple indicator CFA with factor loading invariance
.
.
.
MODEL:      f12a BY bru12
                    fig12 (1)
                    hot12 (2)
                    lie12 (3)
                    stu12 (4)
                    tcl12 (5)
                    yot12 (6);

            f22a BY bru22
                    fig22 (1)
                    hot22 (2)
                    lie22 (3)
                    stu22 (4)
                    tcl22 (5)
                    yot22 (6);
```

247

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance (Continued)

```
MODEL:      f32a BY bru32
                    fig32 (1)
                    hot32 (2)
                    lie32 (3)
                    stu32 (4)
                    tcl32 (5)
                    yot32 (6);

            f42a BY bru42
                    fig42 (1)
                    hot42 (2)
                    lie42 (3)
                    stu42 (4)
                    tcl42 (5)
                    yot42 (6);
```

248

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance

```
TITLE:      Multiple indicator CFA with factor loading and intercept
            incariance
.
.
.
MODEL:      f12a BY bru12
                    fig12 (1)
                    hot12 (2)
                    lie12 (3)
                    stu12 (4)
                    tcl12 (5)
                    yot12 (6);
            f22a BY bru22
                    fig22 (1)
                    hot22 (2)
                    lie22 (3)
                    stu22 (4)
                    tcl22 (5)
                    yot22 (6);
```

249

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance (Continued)

```
MODEL:      f32a BY bru32
                    fig32 (1)
                    hot32 (2)
                    lie32 (3)
                    stu32 (4)
                    tcl32 (5)
                    yot32 (6);

            f42a BY bru42
                    fig42 (1)
                    hot42 (2)
                    lie42 (3)
                    stu42 (4)
                    tcl42 (5)
                    yot42 (6);
```

250

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance (Continued)

```
[bru12 bru22 bru32 bru42] (7);
[fig12 fig22 fig32 fig42] (8);
[hot12 hot22 hot32 hot42] (9);
[lie12 lie22 lie32 lie42] (10);
[stu12 stu22 stu32 stu42] (11);
[tcl12 tcl22 tcl32 tcl42] (12);
[yot12 yot22 yot32 yot42] (13);

[f12a@0 f22a f32a f42a];
```

251

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance And Partial Intercept Invariance

```
TITLE:      Multiple indicator CFA with factor loading and partial
            intercept invariance

MODEL:      f12a BY bru12
                    fig12 (1)
                    hot12 (2)
                    lie12 (3)
                    stu12 (4)
                    tcl12 (5)
                    yot12 (6);
            f22a BY bru22
                    fig22 (1)
                    hot22 (2)
                    lie22 (3)
                    stu22 (4)
                    tcl22 (5)
                    yot22 (6);
```

252

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance And Partial Intercept Invariance (Continued)

```
f32a BY bru32
        fig32 (1)
        hot32 (2)
        lie32 (3)
        stu32 (4)
        tcl32 (5)
        yot32 (6);
f42a BY bru42
        fig42 (1)
        hot42 (2)
        lie42 (3)
        stu42 (4)
        tcl42 (5)
        yot42 (6);
```

253

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance And Partial Intercept Invariance (Continued)

```
[bru12 bru22 bru32 bru42] (7);
[fig12 fig22 fig32 fig42] (8);
[hot12 hot22 hot32      ] (9);
[lie12 lie22 lie32 lie42] (10);
[stu12 stu22            ] (11);
[tcl12 tcl22 tcl32      ] (12);
[yot12 yot22 yot32 yot42] (13);

[f12a@0 f22a f32a f42a];
```

254

Summary of Analysis Results For TOCA Measurement Invariance Models

Model	Chi-Square (d.f.)	Difference (d.f. diff.)
Measurement non-invariance	567.08 (344)	
Factor loading invariance	581.29 (362)	14.21 (18)
Factor loading and intercept invariance	654.59 (380)	73.30* (18)
Factor loading and partial intercept invariance	606.97 (376)	25.68* (14)
Factor loading and partial intercept invariance with a linear growth structure	614.74 (381)	7.77 (5)

Summary of Analysis Results For TOCA Measurement Invariance Models (Continued)

Explanation of Chi-Square Differences

Factor loading invariance	(18)	6 factor loadings instead of 24
Factor loading and intercept invariance	(18)	7 intercepts plus 3 factor means instead of 28 intercepts
Factor loading and partial intercept invariance	(14)	4 additional intercepts
Factor loading and partial intercept invariance with a linear growth structure	(5)	1 growth factor mean instead of 3 factor means 2 growth factor variances, 1 growth factor covariance, 4 factor residual variances instead of 10 factor variances/covariances

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance With A Linear Growth Structure

```
MODEL:      f12a BY bru12
                    fig12 (1)
                    hot12 (2)
                    lie12 (3)
                    stu12 (4)
                    tcl12 (5)
                    yot12 (6);
            f22a BY bru22
                    fig22 (1)
                    hot22 (2)
                    lie22 (3)
                    stu22 (4)
                    tcl22 (5)
                    yot22 (6);
```


Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance With A Linear Growth Structure (Continued)

```
MODEL:      f32a BY bru32
                    fig32 (1)
                    hot32 (2)
                    lie32 (3)
                    stu32 (4)
                    tcl32 (5)
                    yot32 (6);
            f42a BY bru42
                    fig42 (1)
                    hot42 (2)
                    lie42 (3)
                    stu42 (4)
                    tcl42 (5)
                    yot42 (6);
```

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance With A Linear Growth Structure (Continued)

```
[bru12 bru22 bru32 bru42] (7);
[fig12 fig22 fig32 fig42] (8);
[hot12 hot22 hot32      ] (9);
[lie12 lie22 lie32 lie42] (10);
[stu12 stu22            ] (11);
[tcl12 tcl22 tcl32      ] (12);
[yot12 yot22 yot32 yot42] (13);

i s | f12a@0 f22a@1 f32a@2 f42a@3;



Alternative language:

i BY f12a-f42a@1;
s BY f12a@0 f22a@1 f32a@2 f42a@3;
[f12a-f42a@0 i@0 s];
```

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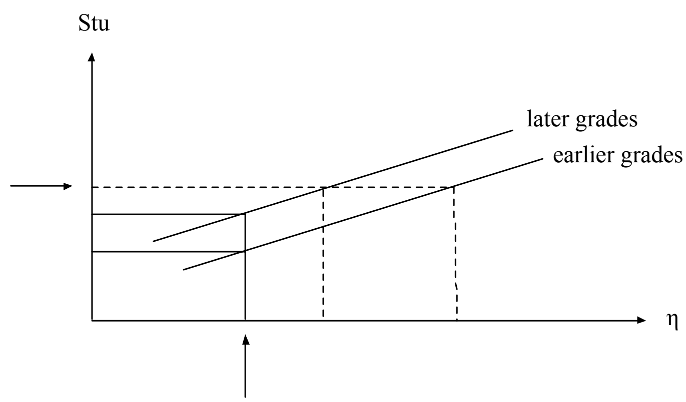
Output Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance With A Linear Growth Structure

	Estimates	S.E.	Est./S.E.	Std	StdYX
F12A |					
BRU12	1.000	.000	.000	.190	.786
FIG12	1.097	.039	28.425	.208	.868
HOT12	.986	.037	26.586	.187	.811
LIE12	.967	.041	23.769	.184	.742
STU12	.880	.041	21.393	.167	.667
TCL12	1.034	.039	26.206	.196	.786
YOT12	.932	.039	23.647	.177	.709
Intercepts					
STU12	.331	.013	25.408	.331	1.324
STU22	.331	.013	25.408	.331	1.231
STU32	.417	.017	24.345	.417	1.592
STU42	.390	.017	23.265	.390	1.496

260

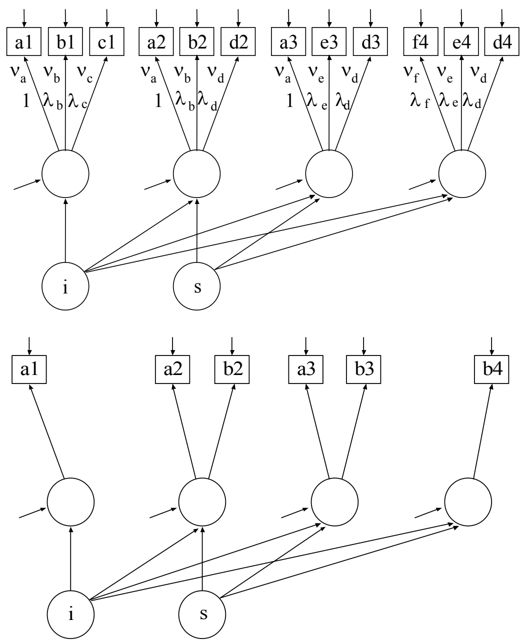


261

Degrees Of Invariance Across Time

- Case 1
  - Same items
  - All items invariant
  - Same construct
- Case 2
  - Same items
  - Some items non-invariant
  - Same construct
- Case 3
  - Different items
  - Some items invariant
  - Same construct
- Case 4
  - Different items
  - Some items invariant
  - Different construct

262

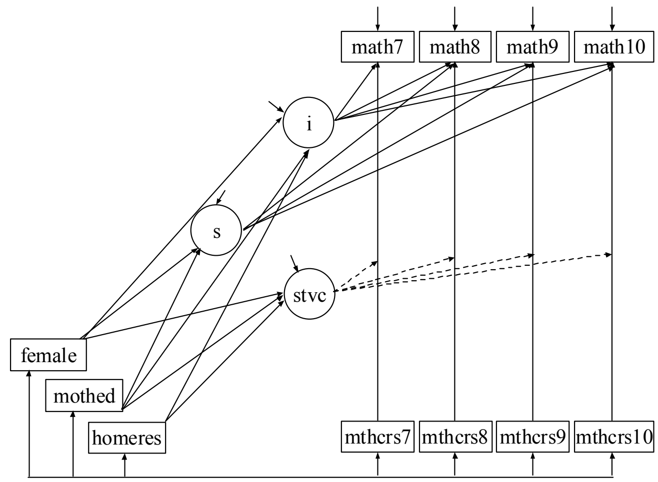


263

Embedded Growth Models

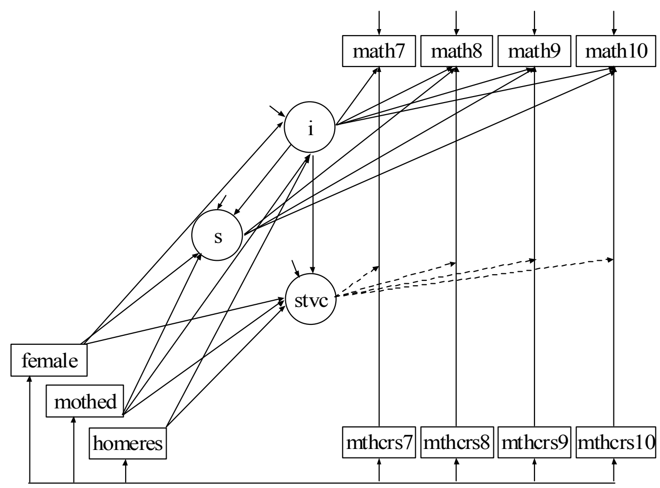
264

Growth Modeling With Time-Varying Covariates



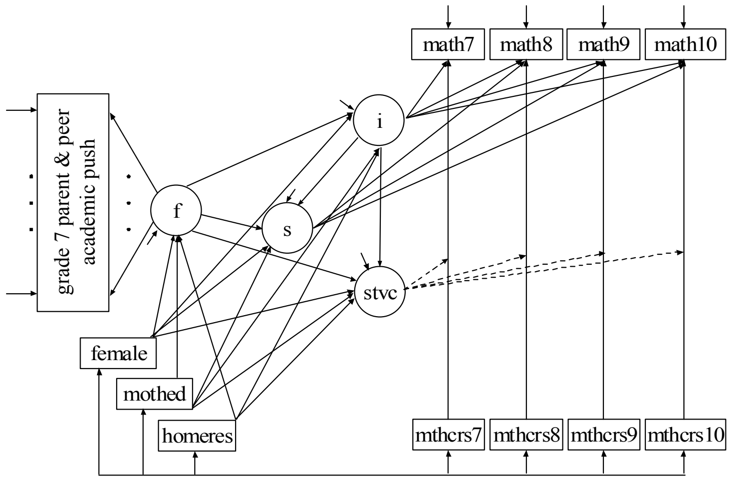
265

A Generalized Growth Model



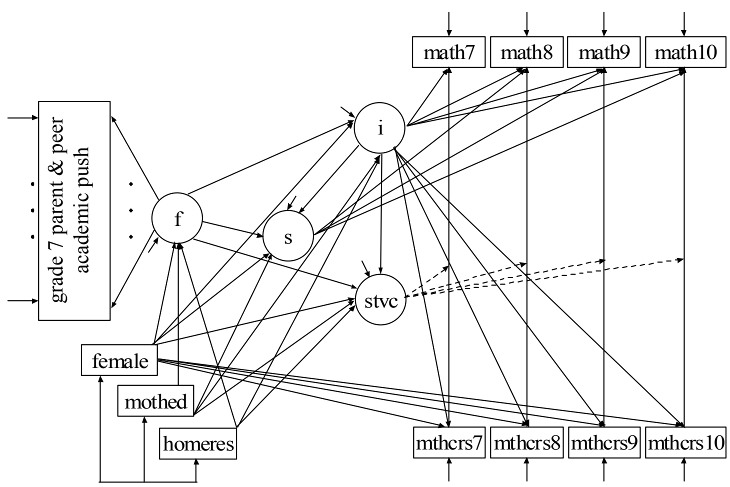
266

A Generalized Growth Model



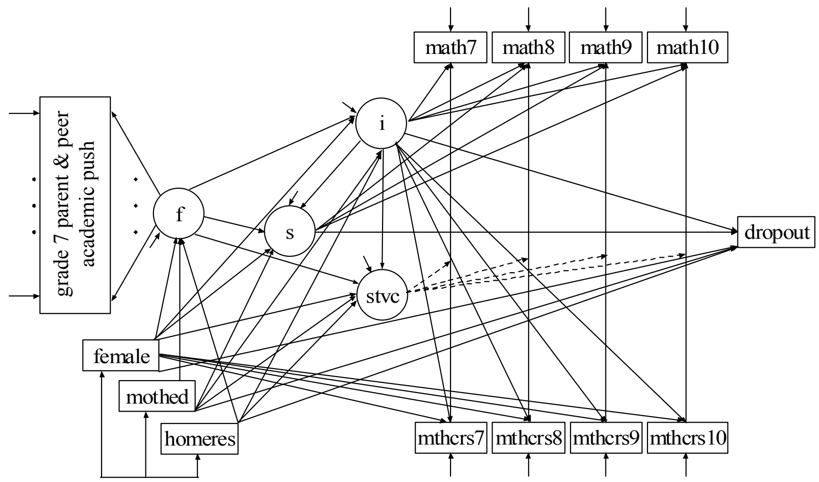
267

A Generalized Growth Model



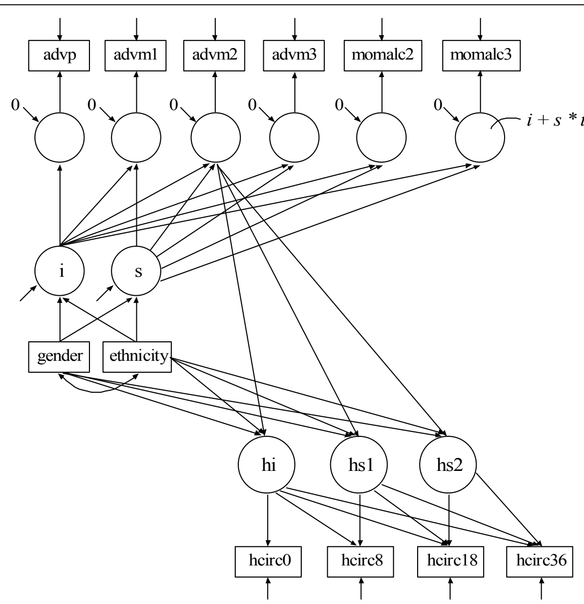
268

A Generalized Growth Model



269

Two Linked Processes



270

Input Excerpts For Two Linked Processes With Measurement Error In The Covariates

```
TITLE:    Embedded growth model with measurement error in the
          covariates and sequential processes
          advp: mother's drinking before pregnancy
          advm1-advm3: drinking in first trimester
          momalc2-momalc3: drinking in 2nd and 3rd trimesters
          hcirc0-hcirc36; head circumference
MODEL:    fadvp    BY  advp;        fadvp@0;
          fadvm1   BY  advm1;      fadvm1@0;
          fadvm2   BY  advm2;      fadvm2@0;
          fadvm3   BY  advm3;      fadvm3@0;
          fmomalc2 BY  momalc2;  fmomalc2@0;
          fmomalc3 BY  momalc3;  fmomalc3@0;
          i BY fadvp-fmomalc3@1;
          s BY fadvp@0  fadvm1@1  fadvm2*2  fadvm3*3
                 fmomalc2-fmomalc3*5 (1);
          [advp-momalc3@0 fadvp-fmomalc3@0 i s];
```

271

Input Excerpts For Two Linked Processes With Measurement Error In The Covariates (Continued)

```
advp WITH advm1; advm1 WITH advm2; advm3 WITH advm2;

i s ON gender eth; s WITH i;


hi   BY  hcirc0-hcirc36@1;
hs1  BY  hcirc0@0 hcirc8@1.196 hcirc36@1.196 hcirc36@1.196;
hs2  BY  hcirc0@0 hcirc8@0 hcirc18@1 hcirc36*2;

[hcirc0-hcirc36@0 hi*34 hs1 hs2];

hs1 WITH hs2@0; hi WITH hs2@0; hi WITH hs1@0;
hi  WITH i@0; hi   WITH s@0; hs1  WITH i@0;
hi1 WITH s@0; hs2  WITH i@0; hs2  WITH s@0;

hi-hs2 ON gender eth fadvm2;
```

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Power For Growth Models

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Designing Future Studies: Power

- Computing power for growth models using Satorra-Saris (Muthén & Curran, 1997; examples)
- Computing power using Monte Carlo studies (Muthén & Muthén, 2002)
- Power calculation web site – PSMG
- Multilevel power (Miyazaki & Raudenbush, 2000; Moerbeek, Breukelen & Berger, 2000; Raudenbush, 1997; Raudenbush & Liu, 2000)
- School-based studies (Brown & Liao, 1999: Principles for designing randomized preventive trials)
- Multiple- (sequential-) cohort power
- Designs for follow-up (Brown, Indurkhia, & Kellam, 2000)

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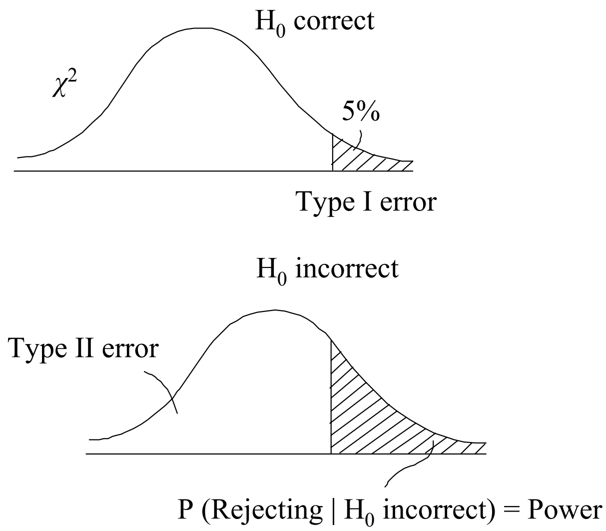
Designing Future Studies: Power



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Power Estimation For Growth Models Using Satorra & Saris (1985)

- Step 1: Create mean vector and covariance matrix for hypothesized parameter values
- Step 2: Analyze as if sample statistics and check that parameter values are recovered
- Step 3: Analyze as if sample statistics, misspecifying the model by fixing treatment effect(s) at zero
- Step 4: Use printed x^2 as an appropriate noncentrality parameter and computer power.

Muthén & Curran (1997): Artificial and real data situations.

276

Input For Step 1 Of Power Calculation

```
TITLE:     Power calculation for a growth model
           Step 1: Computing the population means and
           covariance matrix
DATA:      FILE IS artific.dat;
           TYPE IS MEANS COVARIANCE;
           NOBSERVATIONS = 500;
VARIABLE:  NAMES ARE y1-y4;
MODEL:     i s | y1@0 y2@1 y3@2 y4@3;
           i@.5;
           s@.1;
           i WITH s@0;
           y1-y4@.5;
OUTPUT:    STANDARDIZED RESIUDAL;
```

277

Data For Step 1 Of Power Calculation (Continued)

```
0 0 0 0
1
0 1
0 0 1
0 0 0 1
```

278

Input For Step 2
Of Power Calculation

```
TITLE:     Power calculation for a growth model
           Step 2: Analyzing the population means and
           covariance matrix to check that parameters are
           recovered
DATA:      FILE IS pop.dat;
           TYPE IS MEANS COVARIANCE;
           NOBSERVATIONS = 500;
VARIABLE:  NAMES ARE y1-y4;
MODEL:     i s | y1@0 y2@1 y3@2 y4@3;
OUTPUT:    STANDARDIZED RESIUDAL;
```

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Data For Step 2
Of Power Calculation (Continued)

Data From Step 1 Residual Output

```
0   .2   .4   .6
1
.5   1.1
.5   .7   1.4
.5   .8   1.1   1.9
```

280

Input For Step 3
Of Power Calculation

```
TITLE:     Power calculation for a growth model
           Step 3: Analyzing the population means and
           covariance matrix with a misspecified model
DATA:      FILE IS pop.dat;
           TYPE IS MEANS COVARIANCE;
           NOBSERVATIONS = 50;
VARIABLE:  NAMES ARE y1-y4;
MODEL:     i s | y1@0 y2@1 y3@2 y4@3;
OUTPUT:    STANDARDIZED RESIUDAL;
```

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Step 4 Of Power Calculation

Output Excerpt From Step 3

```
Chi-Square Test of Model Fit

          Value                        9.286
          Degrees of Freedom               6
          P-Value                      .1580
```

Power Algorithm in SAS

```
DATA POWER;
DF=1; CRIT=3.841459;
LAMBDA=9.286;
Power=(1 – (PROBCHI(CRIT, DF, LAMBDA)));
RUN;
```

282

Step 4 Of Power Calculation (Continued)

Results From Power Algorithm

SAMPLE SIZE	POWER
44	0.80
50	0.85
100	0.98
200	0.99

Note: **Non-centrality parameter =**
printed chi-square value from Step 3 =
2*sample size*F

283

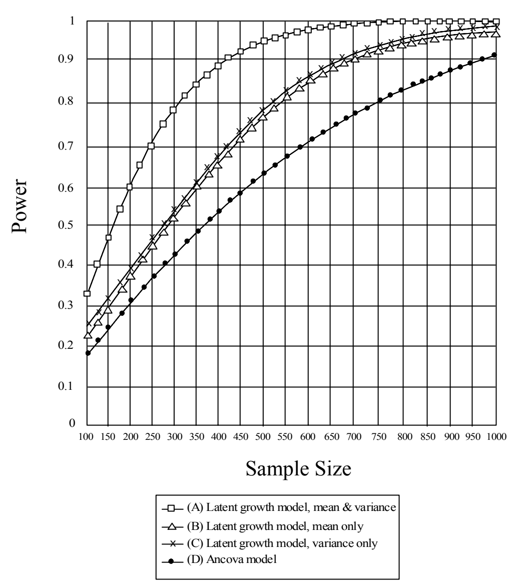

Figure 6. Power to detect a main effect of *ES* = .20 assessed at Time 5.

284

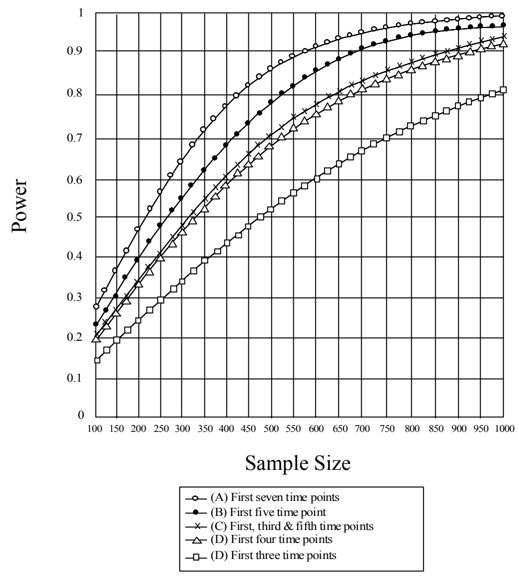

Figure 7. Power to detect a main effect of *ES* = .20 assessed at Time 5 varying as a function of total number of measurement occasions.

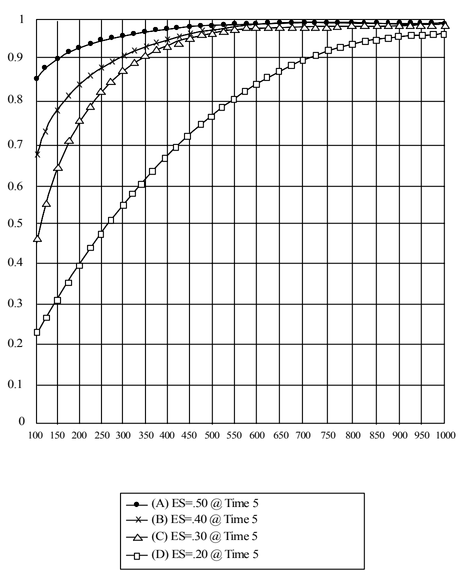

Figure 8. Power to detect various effect sizes assessed at Time 5 based on the first five measurement occasions

Power Estimation For Growth Models Using Monte Carlo Studies

Muthén & Muthén (2002)

Input Power Estimation For Growth Models Using Monte Carlo Studies

```
TITLE:       This is an example of a Monte Carlo
             simulation study for a linear growth model
             for a continuous outcome with missing data
             where attrition is predicted by time-
             invariant covariates (MAR)
MONTECARLO:  NAMES ARE y1-y4 x1 x2;
             NOBSERVATIONS = 500;
             NREPS = 500;
             SEED = 4533;
             CUTPOINTS = x2(1);
             MISSING = y1-y4;
```

Input Power Estimation For Growth Models Using Monte Carlo Studies (Continued)

```
MODEL POPULATION:     x1-x2@1;
                      [x1-x2@0];
                      i s | y1@0 y2@1 y3@2 y4@3;
                      [i*1 s*2];
                      i*1; s*.2; i WITH s*.1;
                      y1-y4*.5;
                      i ON x1*1 x2*.5;
                      s ON x1*.4 x2*.25;
MODEL MISSING:        [y1-y4@-1];
                      y1 ON x1*.4 x2*.2;
                      y2 ON x1*.8 x2*.4;
                      y3 ON x1*1.6 x2*.8;
                      y4 ON x1*3.2 x2*1.6;
```

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Input Power Estimation For Growth Models Using Monte Carlo Studies (Continued)

```
ANALYSIS:     TYPE = MISSING H1;
MODEL:        i s | y1@0 y2@1 y3@2 y4@3;
              [i*1 s*2];
              i*1; s*.2; i WITH s*.1;
              y1-y4*.5;
              i ON x1*1 x2*.5;
              s ON x1*.4 x2*.25;
OUTPUT:       TECH9;
```

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Output Excerpts Power Estimation For Growth Models Using Monte Carlo Studies

Model Results

		ESTIMATES		S.E.	M. S. E.	95%	%Sig
	Population	Average	Std. Dev.	Average		Cover	Coeff
I ON							
X1	1.000	1.0032	0.0598	0.0579	0.0036	0.936	1.000
X2	0.500	0.5076	0.1554	0.1570	0.0241	0.952	0.908
S ON							
X1	0.400	0.3980	0.0366	0.0349	0.0013	0.936	1.000
X2	0.250	0.2469	0.0865	0.0877	0.0075	0.938	0.830

Cohort-Sequential Designs and Power

Considerations:

- Model identification
- Number of timepoints needed substantively
- Number of years of the study
- Number of cohorts: More gives longer timespan but greater risk of cohort differences
- Number of measurements per individual
- Number of individuals per cohort
- Number of individuals per age

Tentative conclusion:

Power most influenced by total timespan, not the number of measures per cohort

References

(To request a Muthén paper, please email bmuthen@ucla.edu.)

Analysis With Longitudinal Data

Introductory

Bollen, K.A. & Curran, P.J. (2006). Latent curve models. A structural equation perspective. New York: Wiley.

Collins, L.M. & Sayer, A. (Eds.) (2001). New methods for the analysis of change. Washington, D.C.: American Psychological Association.

Curran, P.J. & Bollen, K.A. (2001). The best of both worlds: Combining autoregressive and latent curve models. In Collins, L.M. & Sayer, A.G. (Eds.) New methods for the analysis of change (pp. 105-136). Washington, DC: American Psychological Association.

Duncan, T.E., Duncan, S.C., Strycker, L.A., Li, F., & Alpert, A. (1999). An introduction to latent variable growth curve modeling: concepts, issues, and applications. Mahwah, NJ: Lawrence Erlbaum Associates.

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