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Cross-lag Mean Structure Models vs Latent Curve Structural Equation Models of Change

Peter M. Bentler

University of California, Los Angeles

OUTLINE

Introduction

A Hierarchy of Modeling Challenges

Longitudinal Models as Covariance Structures Only

- Comment on difference scores, latent residual gain
- Drug use examples

Mean and Covariance Structures

Mean/Covariance Autoregressive Models

Mean/Covariance Growth Curve Models

A Study on the Gateway Hypothesis of Drug Use

A Hierarchy of Modeling Challenges

1. A single group covariance structure model

   e.g., $\Sigma = \Lambda\Phi\Lambda' + \Psi$

2. A single group covariance structure model with fixed a priori loadings

   e.g., simplex structure $\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

   e.g., circumplex structure $\Lambda = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

   e.g., initial status growth curve model

   $$\Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{bmatrix}$$

3. Single Sample Mean Structure Model

(a) $\mu = \mu(\theta)$, Σ free [note: may be easier than CSA]

(b) $\mu = \mu(\theta)$, $\Sigma = \Sigma(\gamma)$ (no fixed nonzero parameters)

(c) $\mu = \mu(\theta)$, $\Sigma = \Sigma(\theta)$ (no fixed nonzero parameters)

(d) $\mu = \mu(\theta)$, $\Sigma = \Sigma(\theta)$ (many fixed nonzero parameters)

Example of (d): Standard Growth Curve Model

$\mu = \Lambda\mu_{\xi}$, $\Sigma = \Lambda\Phi\Lambda' + \Psi$, Λ all fixed parameters

4. Multiple Sample Covariance Structures with cross-group constraints

5. Multiple Sample Mean and Covariance Structures with cross-group constraints

6. Multiple Sample Mean and Covariance Structures with cross-group constraints, including fixed nonzero parameters (e.g., growth curves in 2 or more samples)

7. Mixture variants of #6, that is, multiple mean and covariance structures based on unknown and probabilistic group membership

8. Now add a multilevel or hierarchical structure (e.g., Within and Between Cluster models) to each of these, etc.

Longitudinal Models as Covariance Structures Only

A comment on difference scores – factors may disappear

$$x_1 = \Lambda\xi_1 + \varepsilon_1$$
$$x_2 = \Lambda\xi_2 + \varepsilon_2 \text{ (Note same } \Lambda\text{)}$$

This means that the difference score has structure

$$x_2 - x_1 = \left(\Lambda\xi_2 + \varepsilon_2\right) - \left(\Lambda\xi_1 + \varepsilon_1\right)$$
$$= \Lambda(\xi_2 - \xi_1) + (\varepsilon_2 - \varepsilon_1)$$

So if the factor scores stay quite stable, the variance of the difference factor $(\xi_2 - \xi_1)$ may get quite small. Most of the variance in $(x_2 - x_1)$ may be due to $(\varepsilon_2 - \varepsilon_1)$.

An underused alternative is latent variable residual gain (see Newcomb, Scheier, & Bentler, 1993)

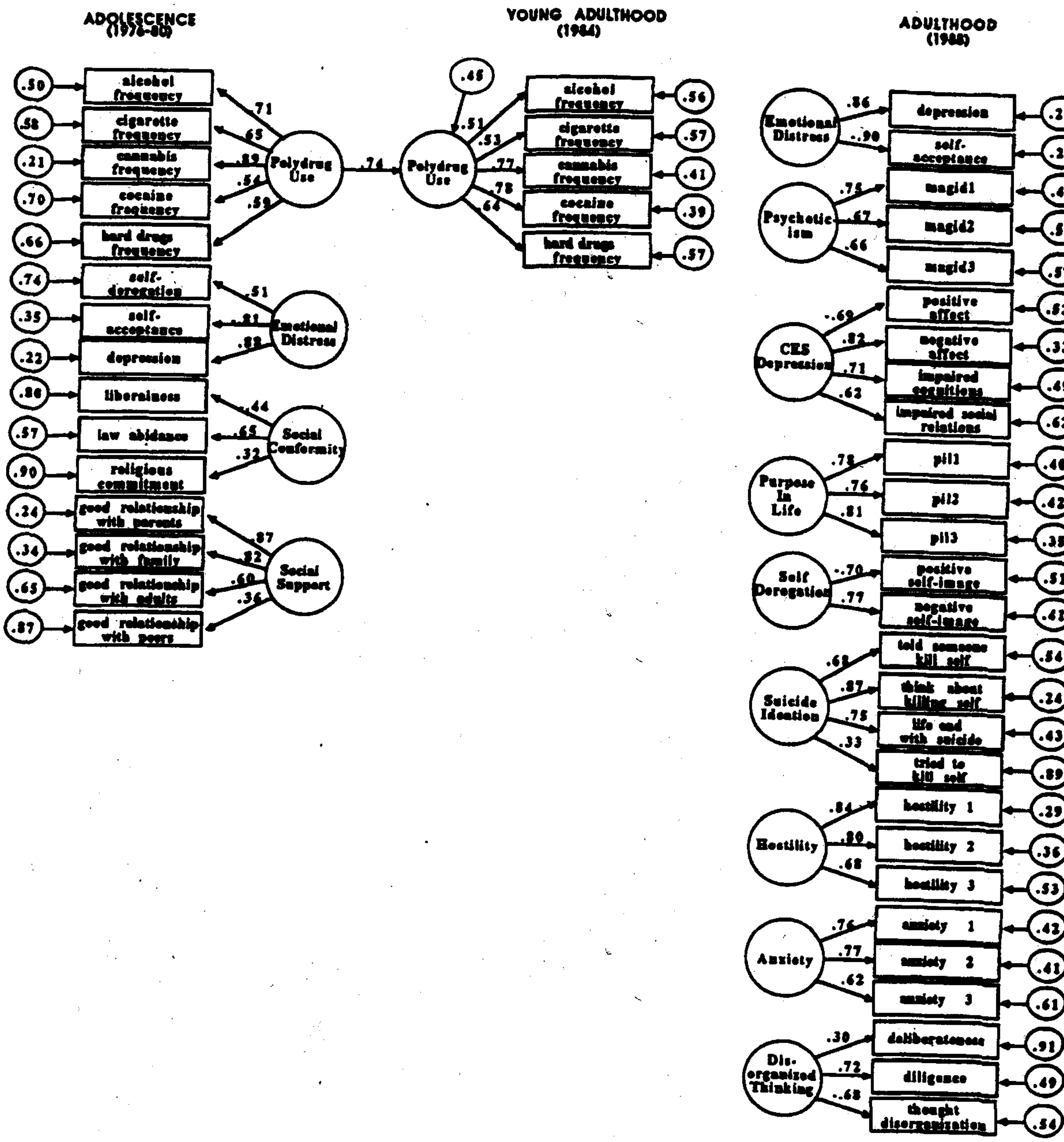
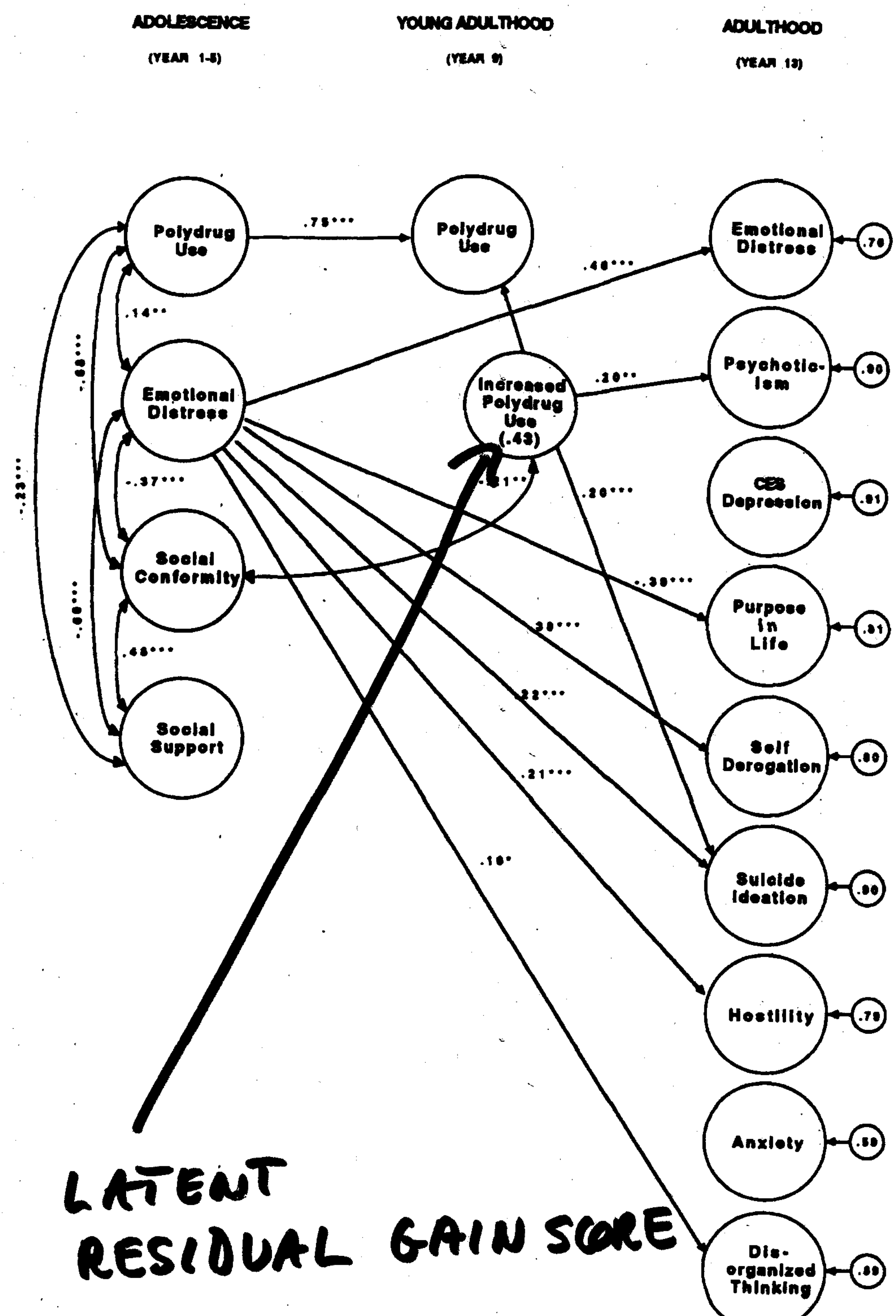


Figure 1. Final confirmatory factor analysis model. (Large circles represent latent constructs, rectangles are measured variables, and small circles with numbers are residual variables. Factor loadings are standardized and all are significant [$p < .001$]. Not depicted in the figure are two-headed arrows [i.e., correlations] joining each possible pair of factors. Estimates for these correlations are given in Table 3. CES = Center for Epidemiologic Studies; magid = magic ideation; pil = purpose in life.)

tors to the various adult mental health measures (both latent and observed). These paths are all from the final structural model and must be considered in concert with results from the same model given in Figure 2 and Table 4. These specific or nonstandard paths provide a detailed picture of more specific influences of both drug and nondrug adolescent variables on adult mental health (see



RES(=D2) = POLY2(F2) − .75*POLY1(F1)

OR POLY2(F2) = .75*POLY1(F1) + D2

WEIGHTED LATENT DIFFERENCE SCORE

__Consequences of Adolescent Drug Use: Impact on the Lives of Young Adults__

by Michael D. Newcomb & Peter M. Bentler

(Newbury Park: Sage Publications, 1988)

Jr. High Students Followed to Adulthood

Natural Variation in Drug Use

Many Initial Control Variables

Latent Variable Methodology

Nonstandard (non-LISREL) models

Consequences on: FAMILY FORMATION AND STABILITY; CRIMINALITY AND DEVIANT BEHAVIOR; SEXUAL BEHAVIOR AND INVOLVEMENT; EDUCATIONAL PURSUITS; LIVELIHOOD PURSUITS; MENTAL HEALTH; SOCIAL INTEGRATION

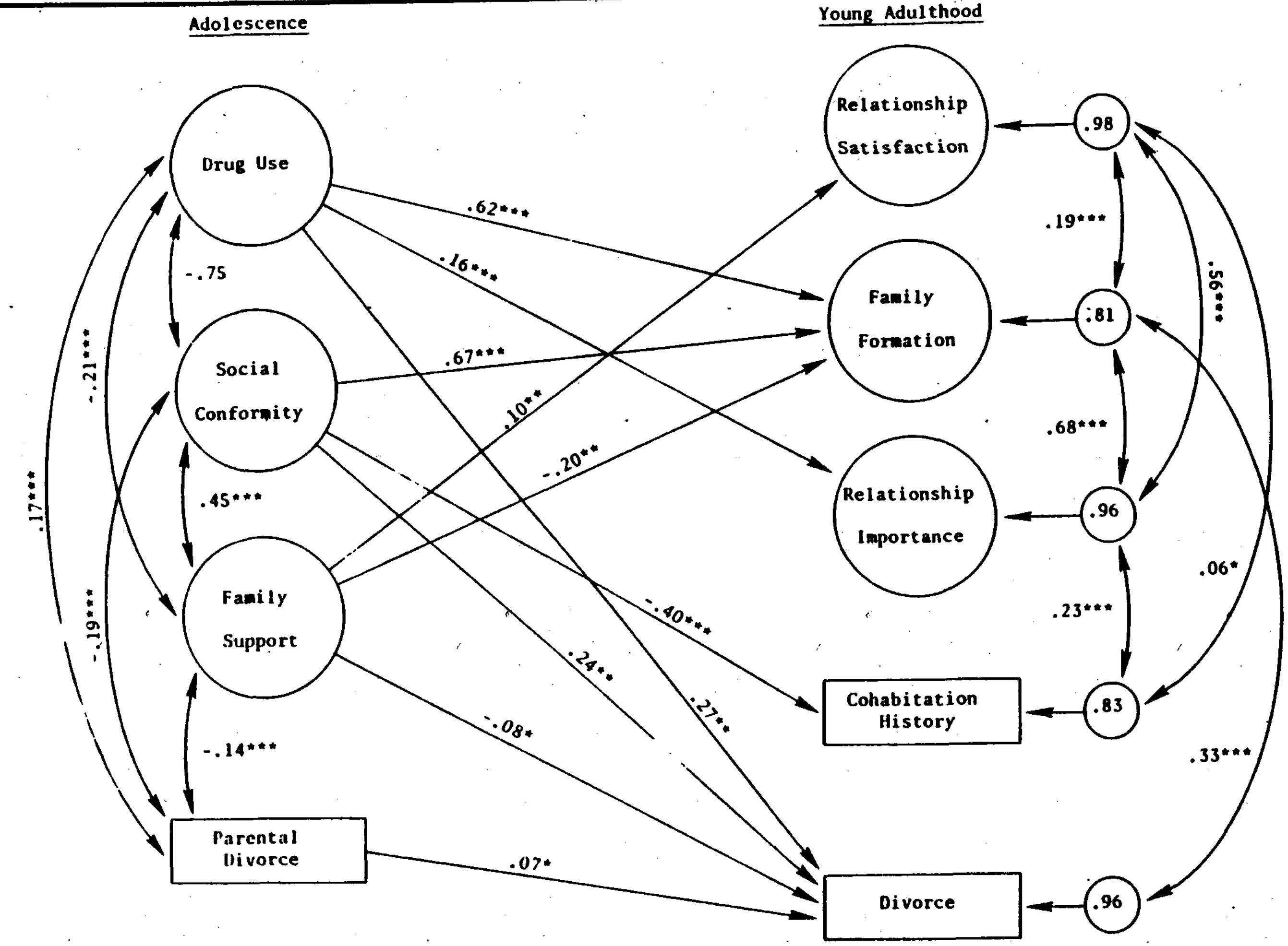


Figure 7.2. Final Structural or Path Model for the Family Formation Variables

NOTE: The measurement portion of this model is not depicted for simplicity. Two-headed arrows reflect correlations and single-headed arrows represent across-time regression effects. Parameter estimates are standardized, residual variables are variances, and significance levels were determined by critical ratios (*$p < .05$; **$p < .01$; ***$p < .001$). Nonstandard effects for this model are given in Table 7.2.

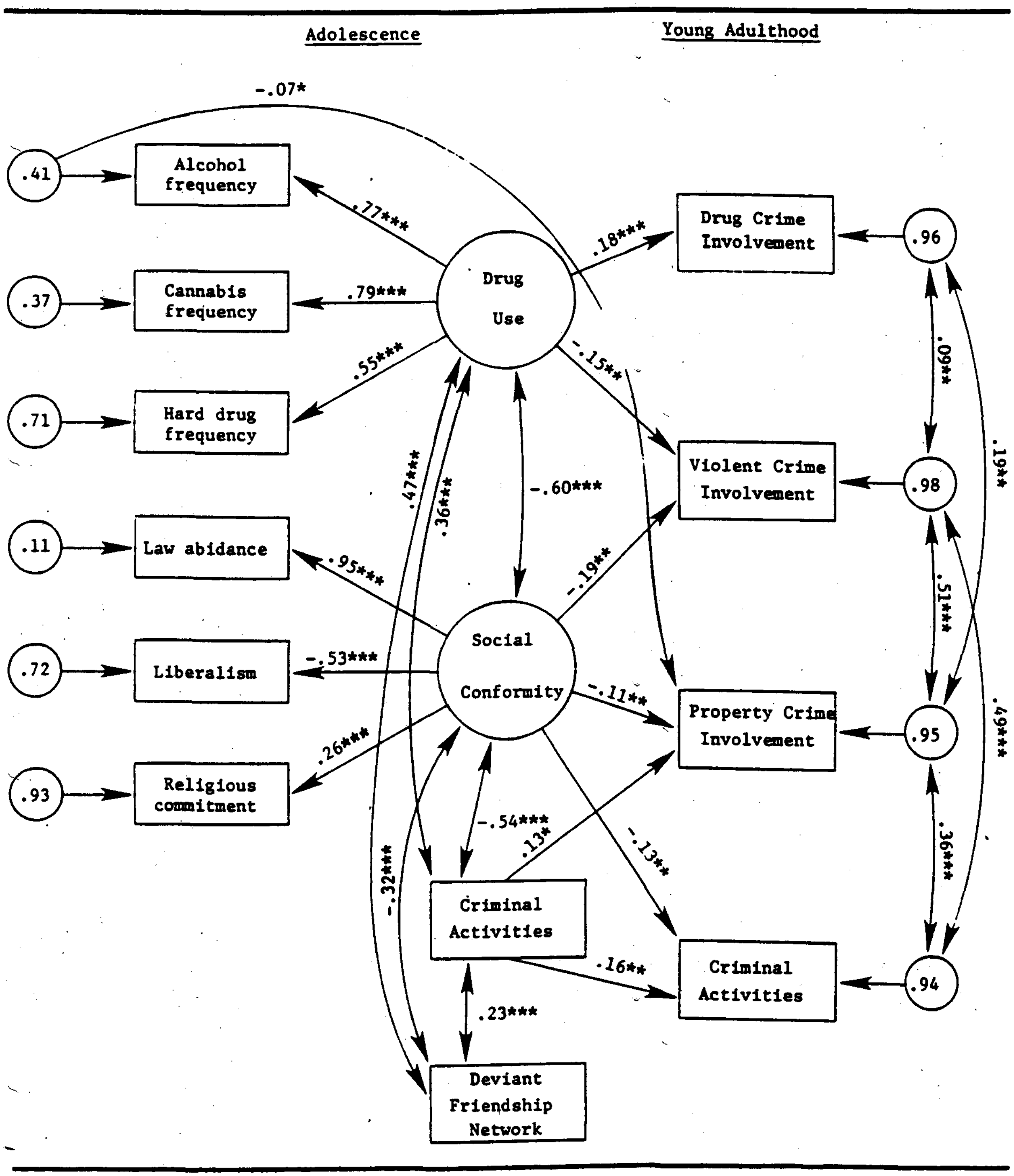


Figure 8.3. Final Structural Model Including Measurement Model for the Distribution-Free Estimates (AGLS) of the Deviant Behavior Variables

NOTE: Large circles reflect latent factors, rectangles represent measured variables, and small circles with numbers are residual variances. Two-headed arrows are correlations and single-headed arrows represent regression effects. Parameter estimates are standardized and significance levels were determined by critical ratios (*p < .05; **p < .01; ***p < .001).

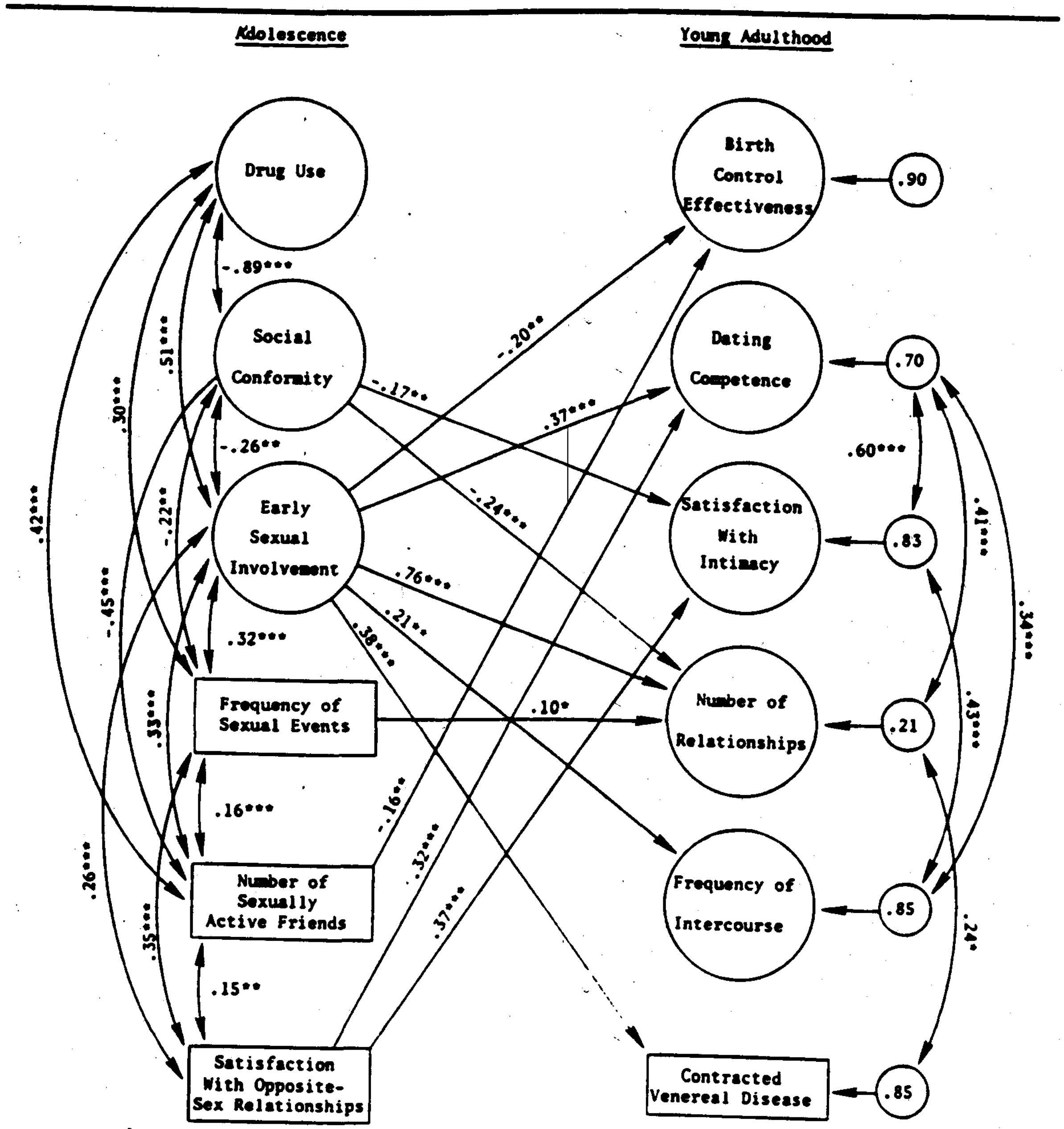
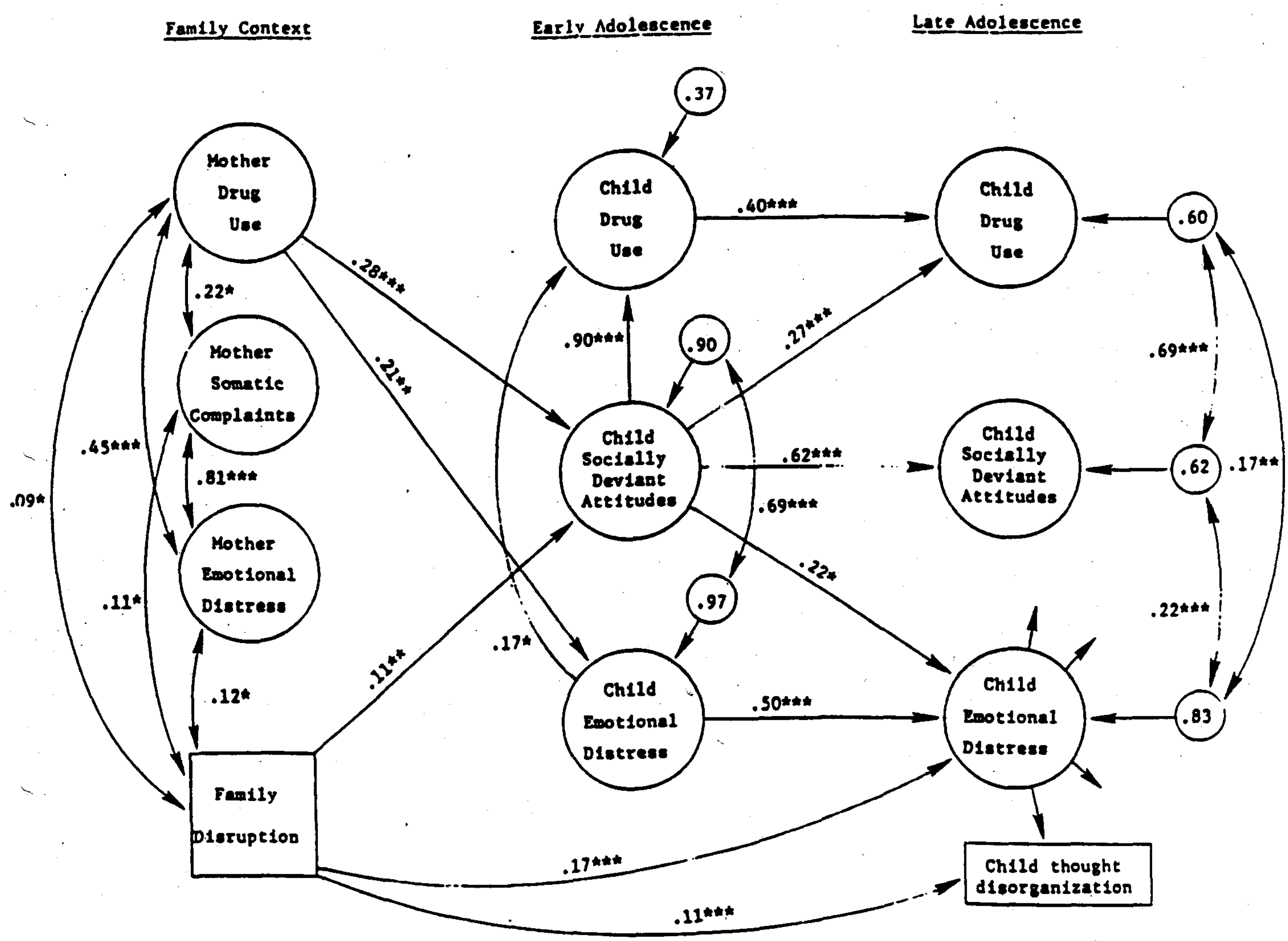
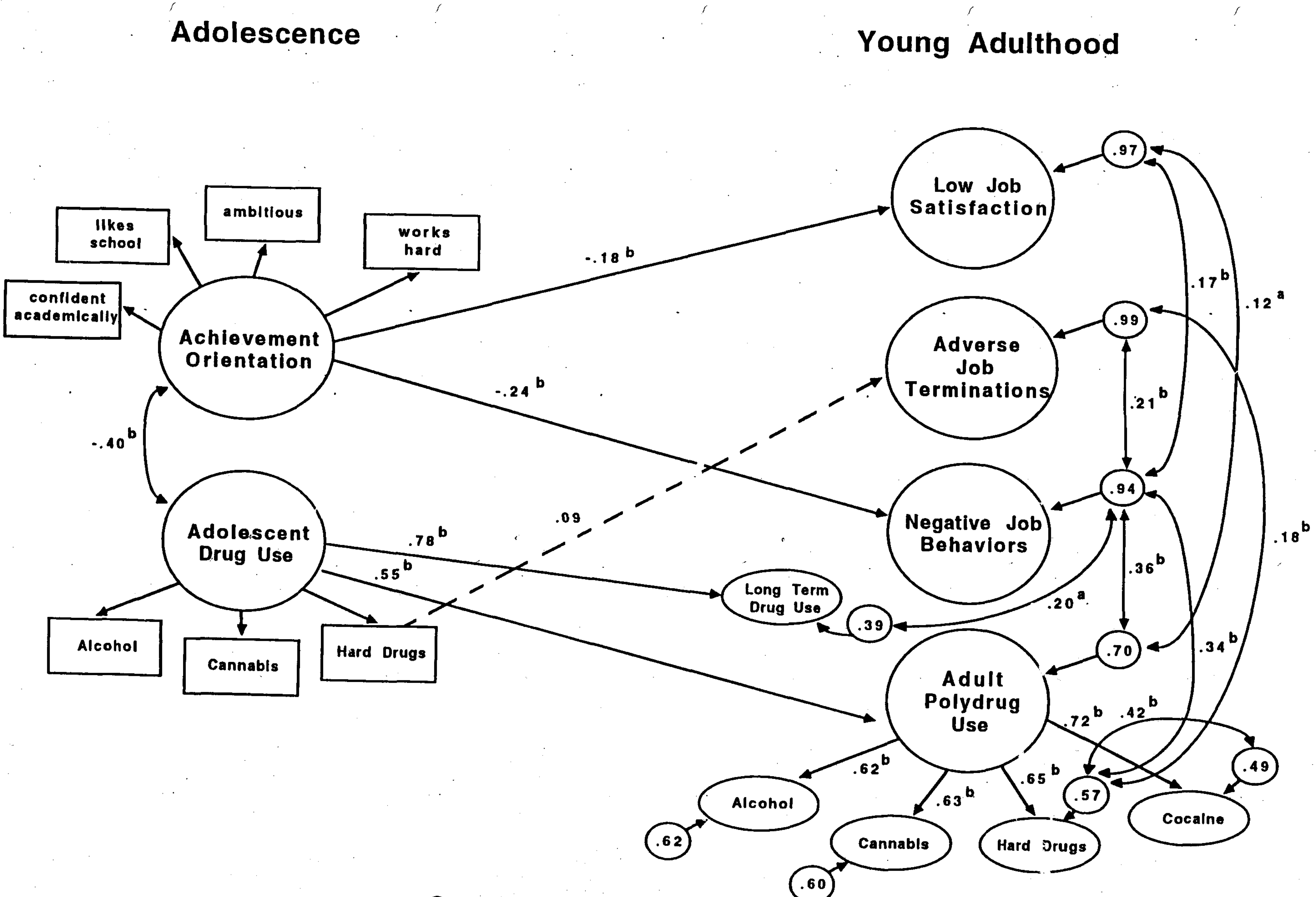


Figure 9.4. Final Structural or Path Model for the Men's Sexual Behavior Variables

NOTE: The measurement portion of this model is not depicted for simplicity. Two-headed arrows reflect correlations and single-headed arrows represent across-time regression effects. Parameter estimates are standardized, residual variables are variances, and significance levels were determined by critical ratios (*p < .05; **p < .01; ***p < .001). Nonstandard effects for this model are given in Table 9.4.

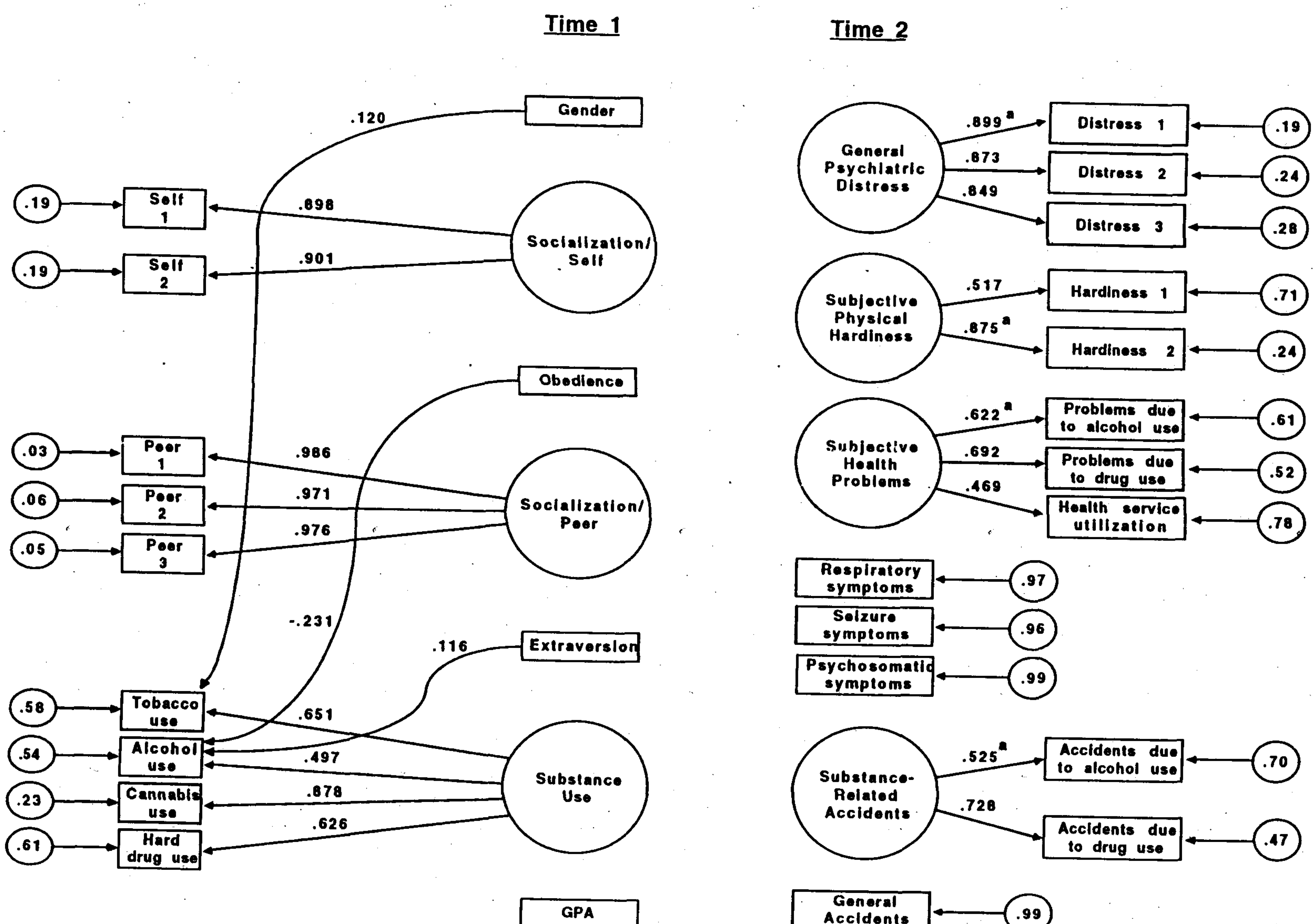


Newcomb, M. D., & Bentler, P. M. (1988). The impact of family context, deviant attitudes, and emotional distress on adolescent drug use: Longitudinal latent-variable analyses of mothers and their children. *Journal of Research in Personality*, 22, 154-176.



STEIN, SMITH, GUY, & BENTLER
J APPLIED PSYCH., (1993)

Figure :



GUY, SMITH, & BENTLER, 1993 Psychology & Health

Figure 2

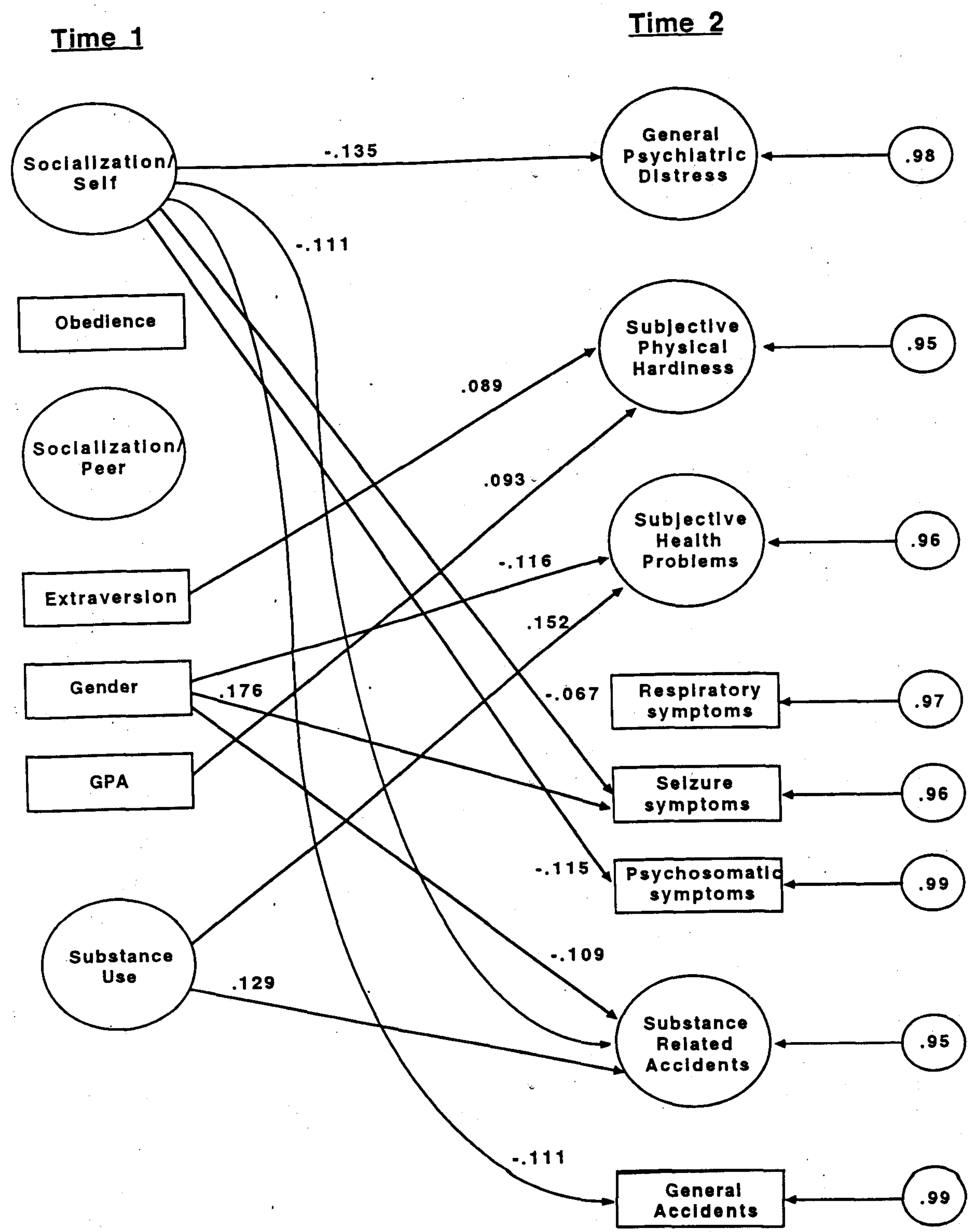
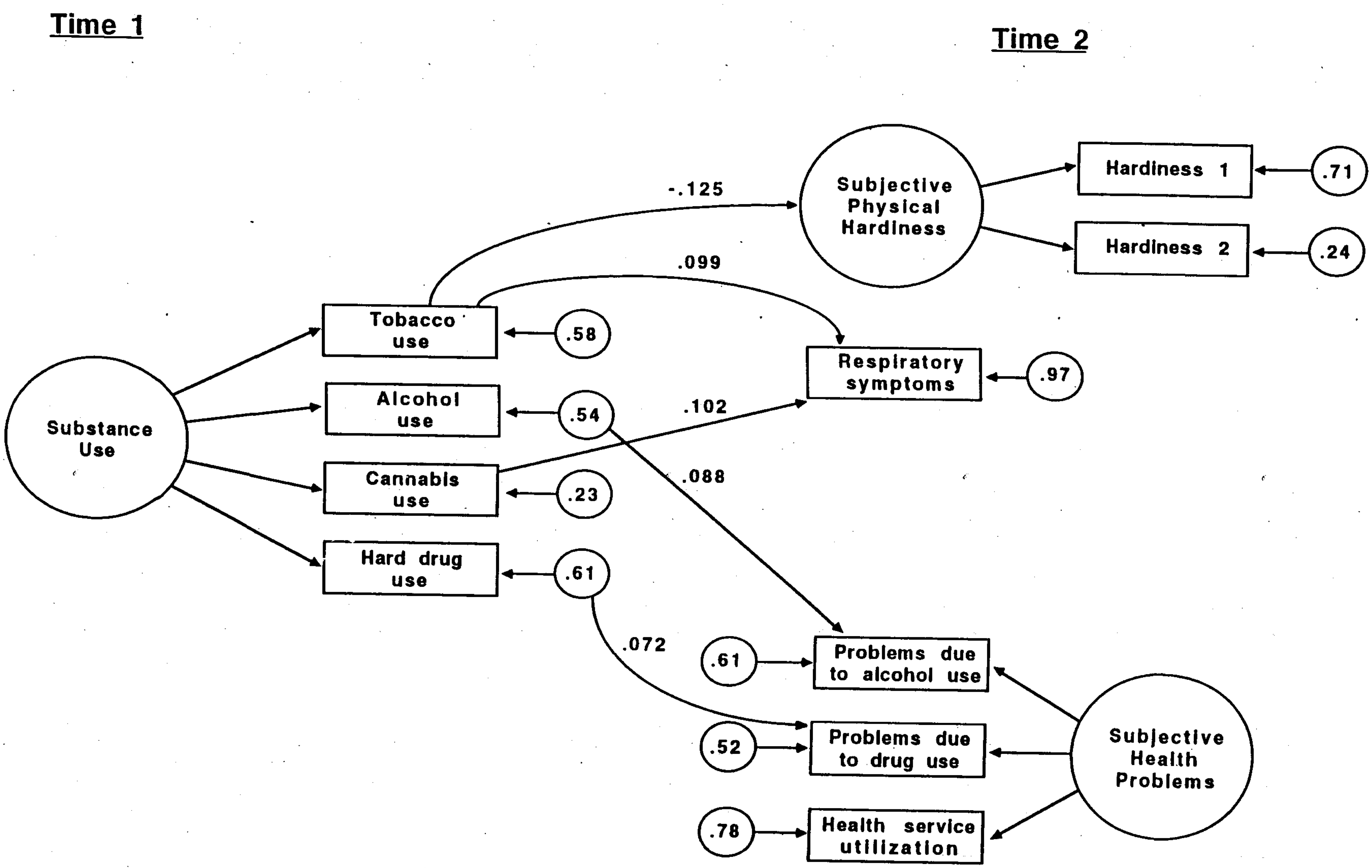
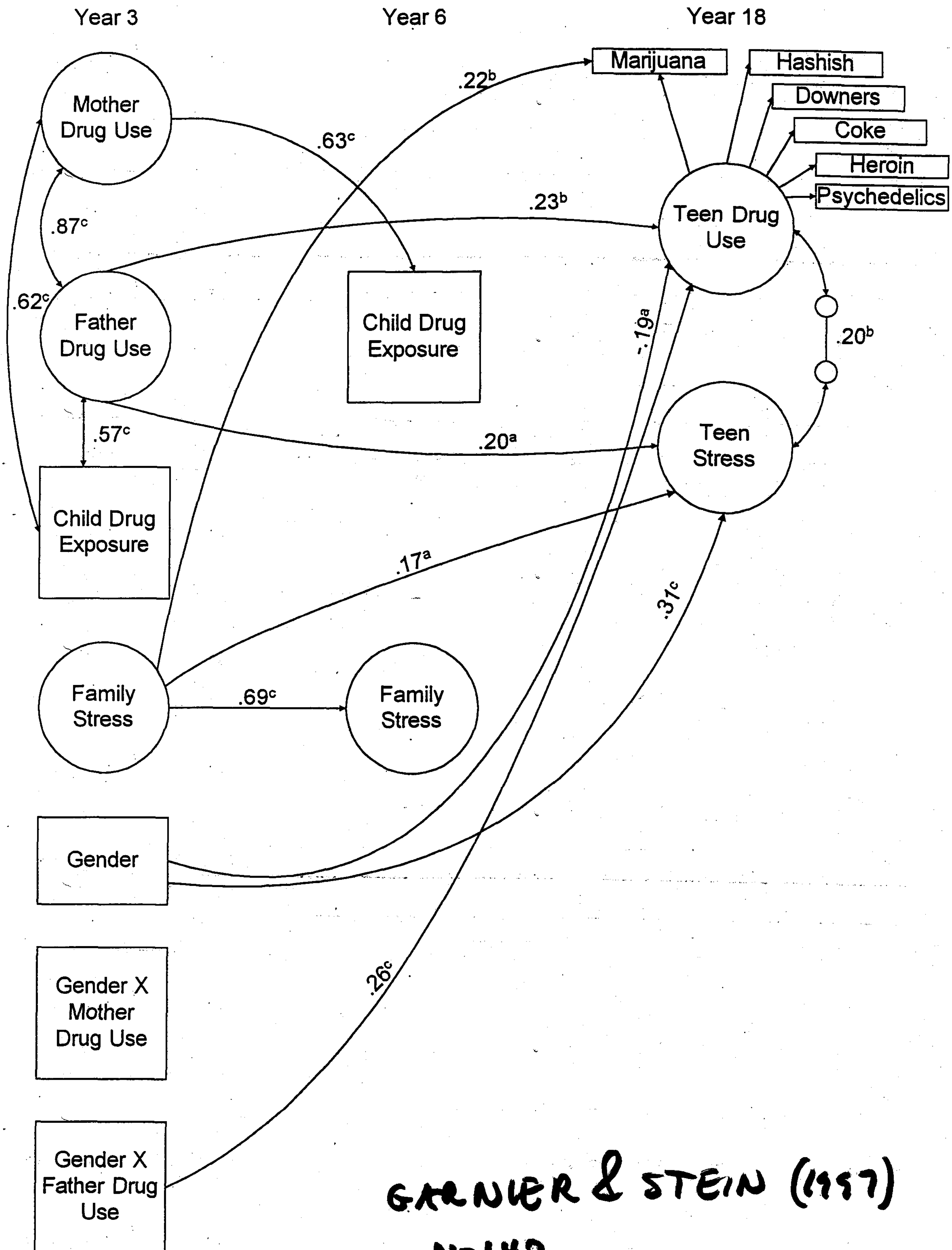

Time 1
Time 2
Socialization/ Self
Obedience
Socialization/ Peer
Extraversion
Gender
GPA
Substance Use
General Psychiatric Distress
Subjective Physical Hardiness
Subjective Health Problems
Respiratory symptoms
Seizure symptoms
Psychosomatic symptoms
Substance Related Accidents
General Accidents
-.135
-.111
.089
.093
-.116
.152
-.067
.176
-.115
-.109
.129
-.111
.98
.95
.96
.97
.96
.99
.95
.99

Figure 3

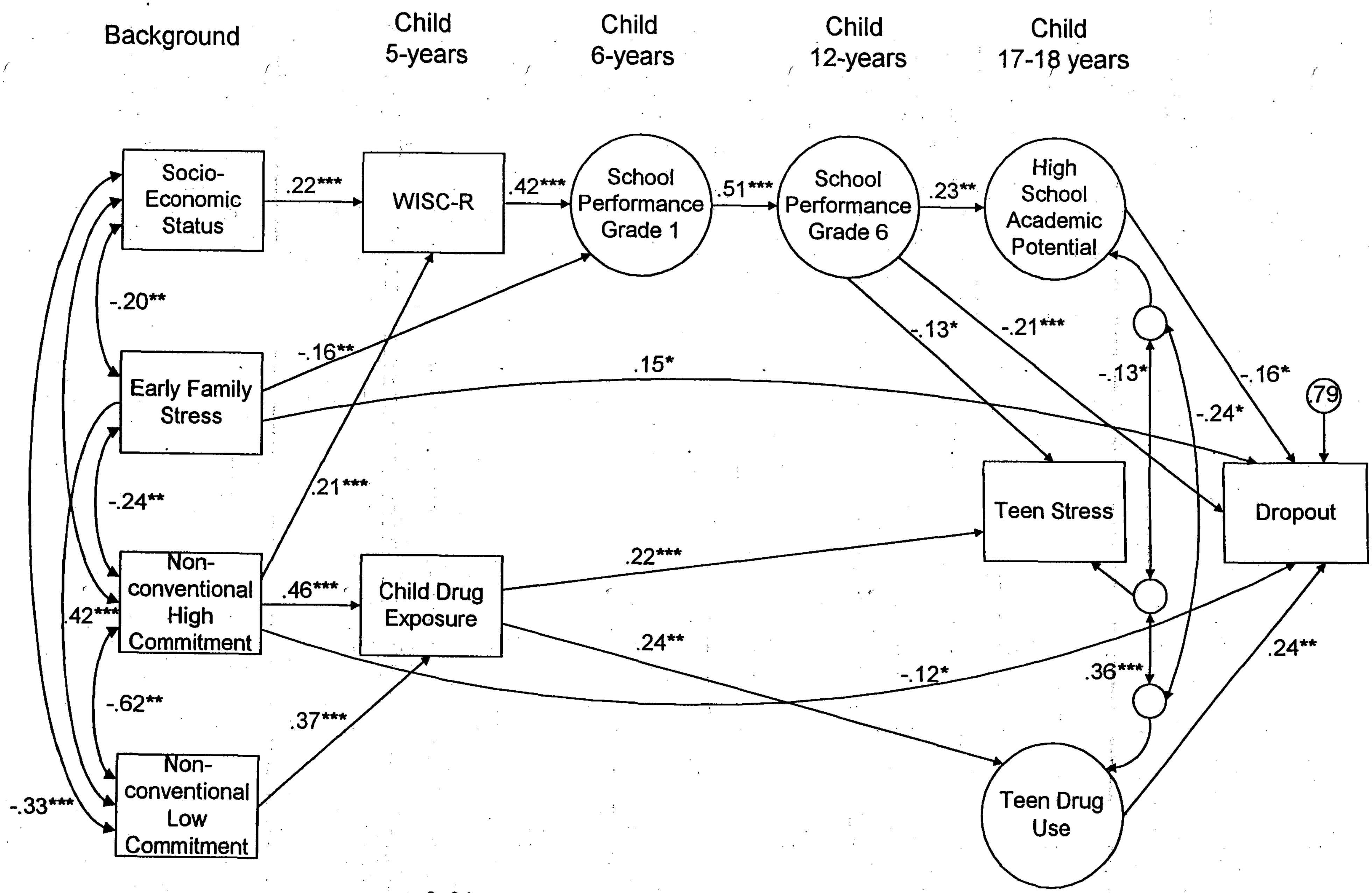


Time 1
Time 2
Substance Use
Tobacco use
Alcohol use
Cannabis use
Hard drug use
.58
.54
.23
.61
-.125
.099
.102
.088
.072
Subjective Physical Hardiness
Hardiness 1
Hardiness 2
.71
.24
Respiratory symptoms
.97
Problems due to alcohol use
Problems due to drug use
Health service utilization
.61
.52
.78
Subjective Health Problems



GARNIER & STEIN (1997)

N=147



N= 194

GARNIER, STEIN, & JACOBS (1997)

The Addiction-Prone Personality

Gordon E. Barnes
University of Victoria, Victoria, British Columbia, Canada

Robert P. Murray and **David Patton**
University of Manitoba, Winnipeg, Manitoba, Canada

Peter M. Bentler
University of California, Los Angeles, California

and

Robert E. Anderson
University of New Mexico, Albuquerque, New Mexico

Kluwer Academic/Plenum Publishers
New York, Boston, Dordrecht, London, Moscow
(2000)

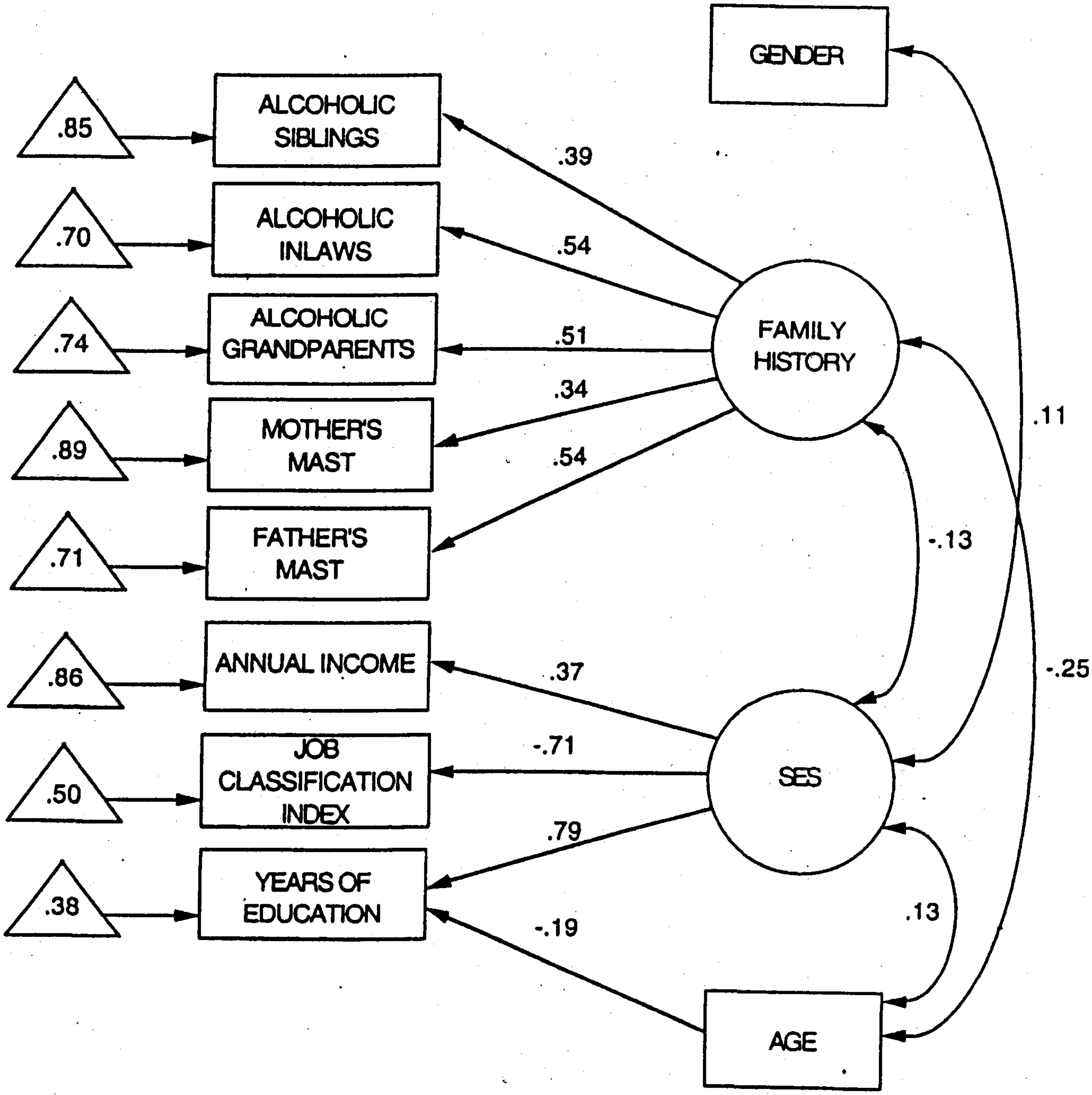


Figure 5.2. Factor loadings of background variables for the longitudinal structural mod

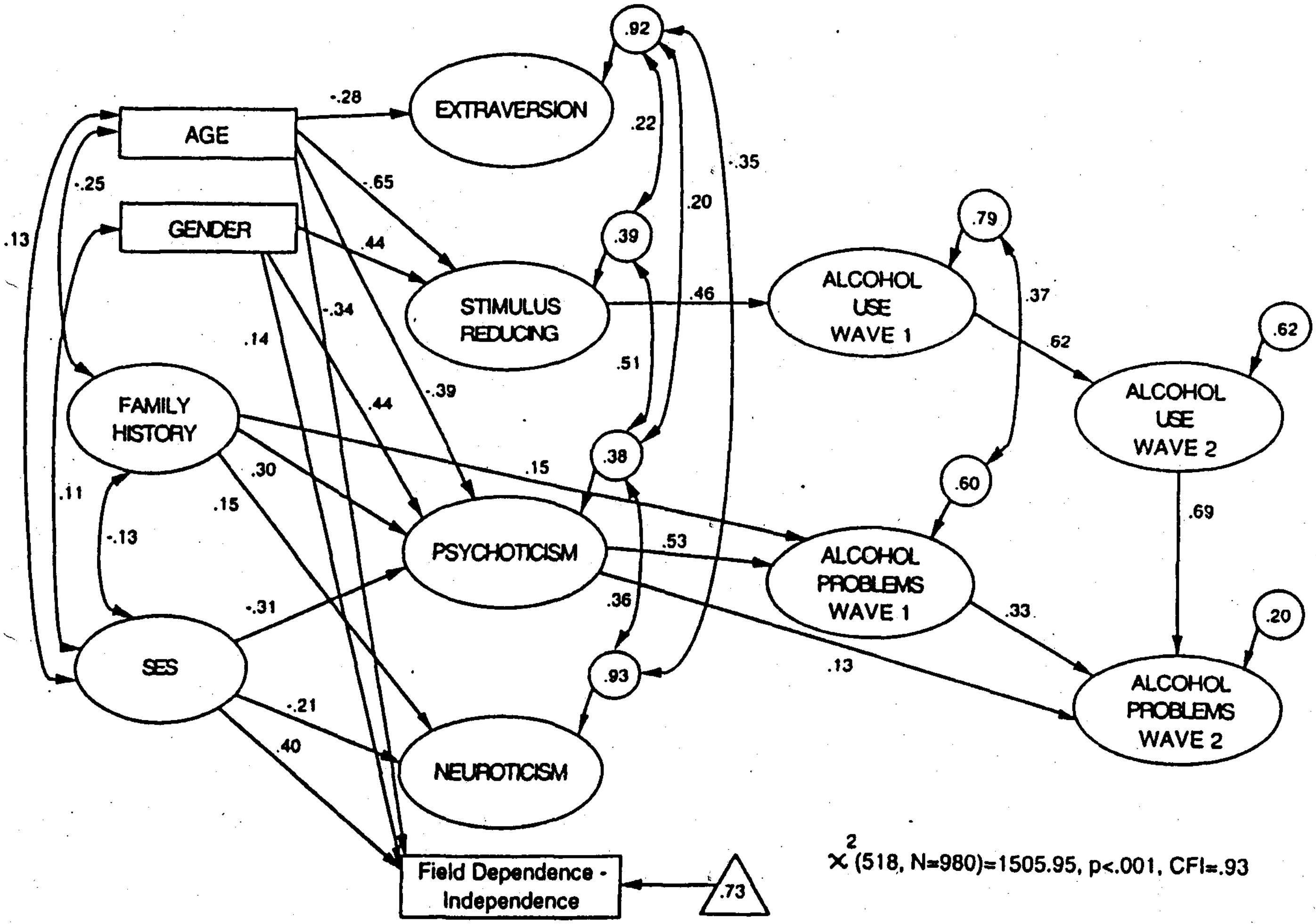


Figure 5.1. Structural equation longitudinal model: relationships among demographics, personality, and drinking.

Prevention Science, Vol. 3, No. 2, June 2002 (© 2002)

Positive Impact of Competence Skills and Psychological Wellness in Protecting Inner-City Adolescents From Alcohol Use

Jennifer A. Epstein,[1,2] Kenneth W. Griffin,[1] and Gilbert J. Botvin[1]

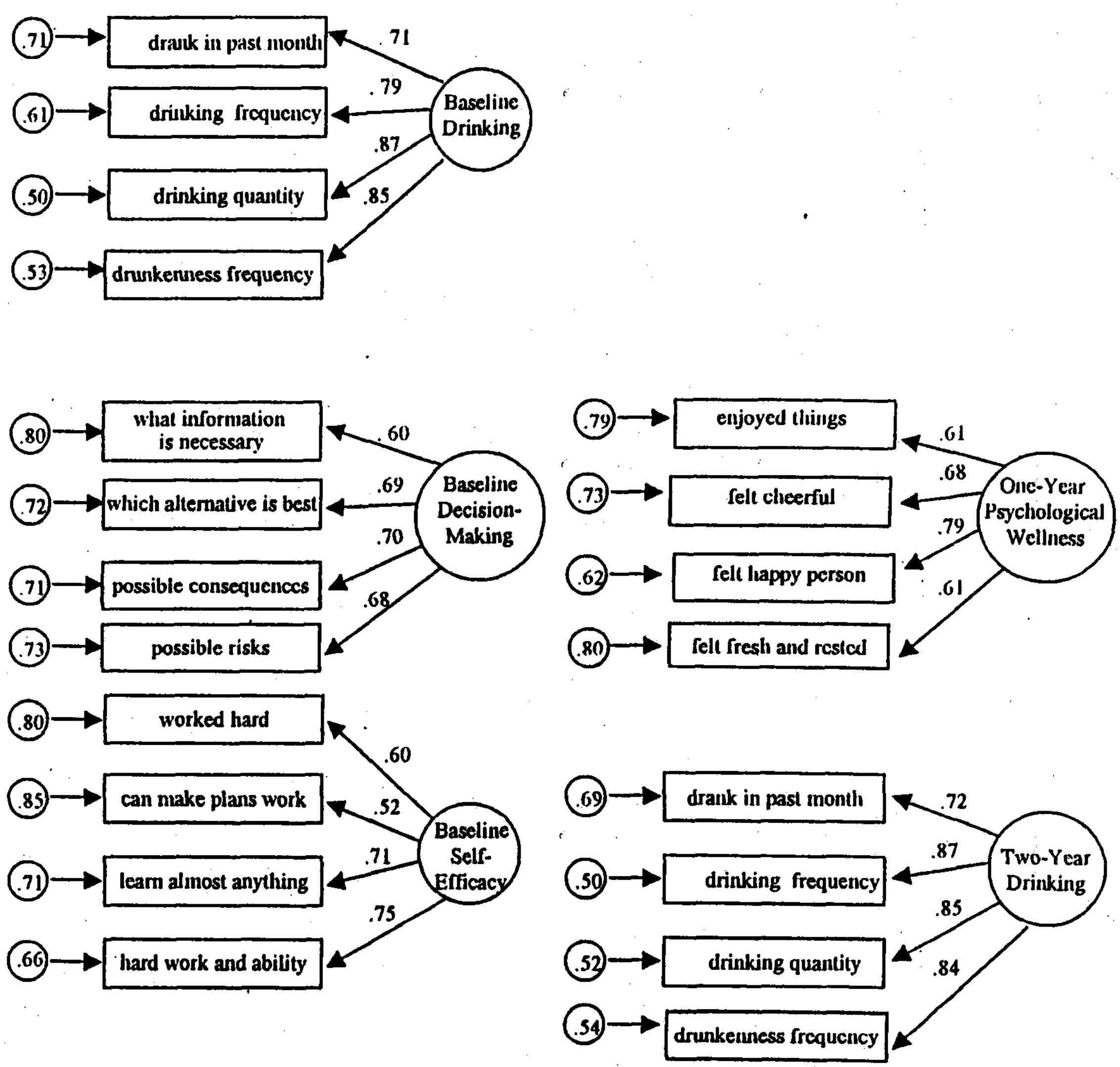


Fig. 1. Confirmatory factor analysis model. Large circles represent latent constructs, rectangles are measured variables, and single-headed arrows designate residual variances.

Wellness Protects Youth From Alcohol Use   Epstein, Griffin & Botvin (2002)

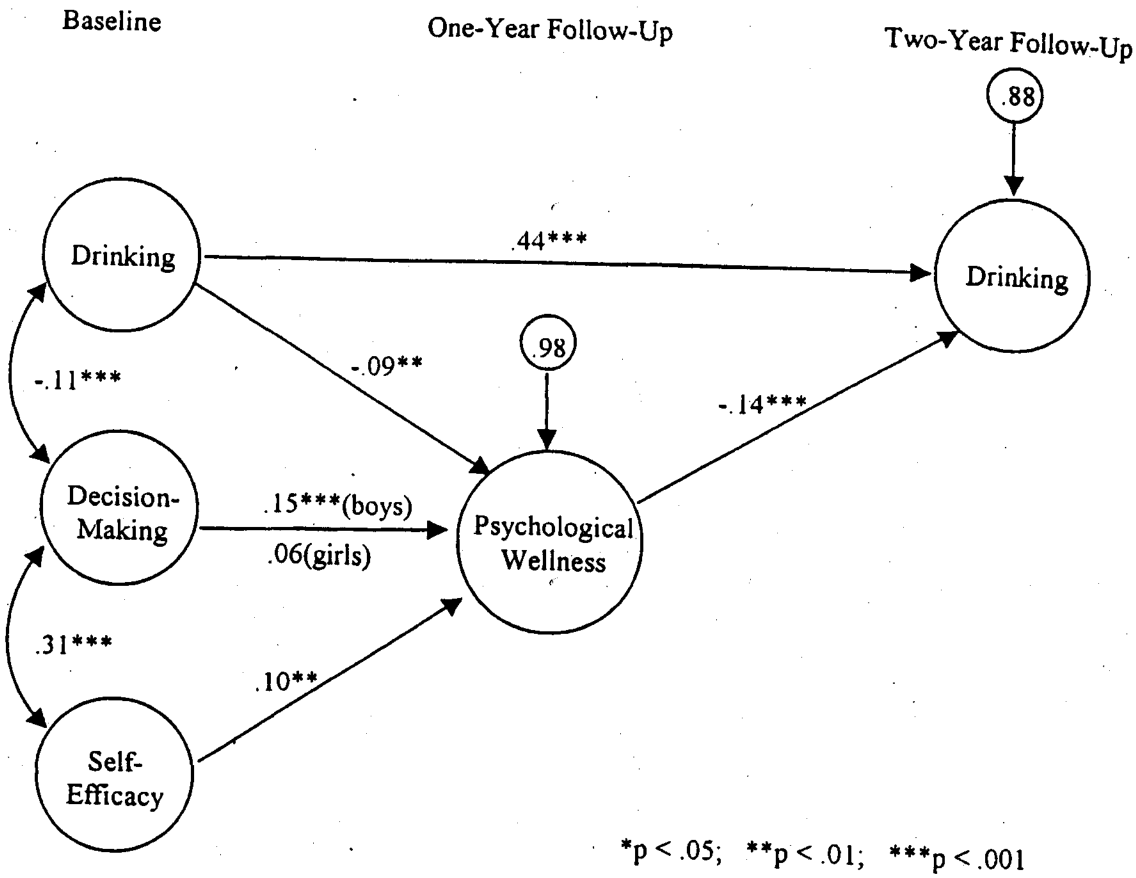
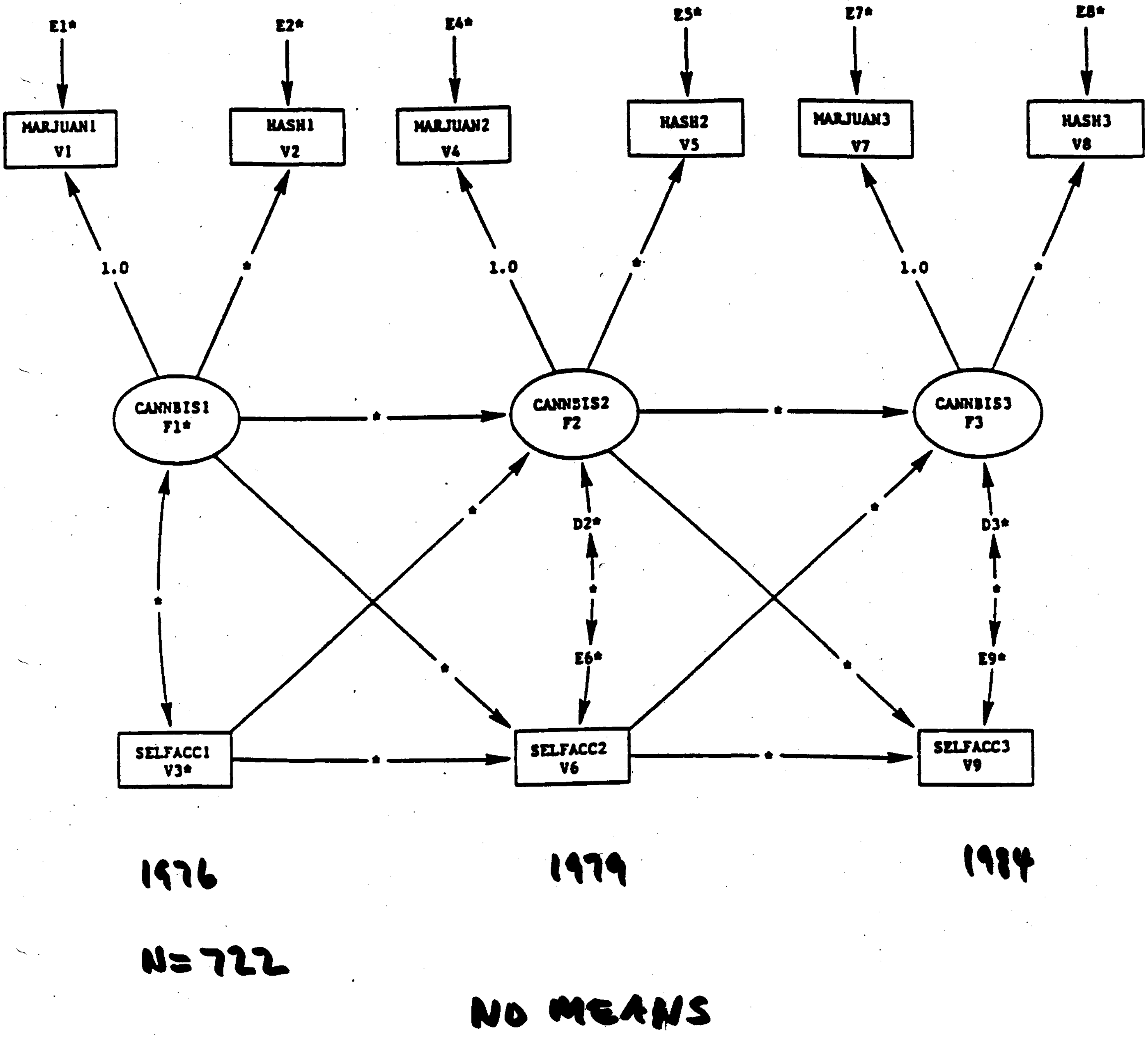
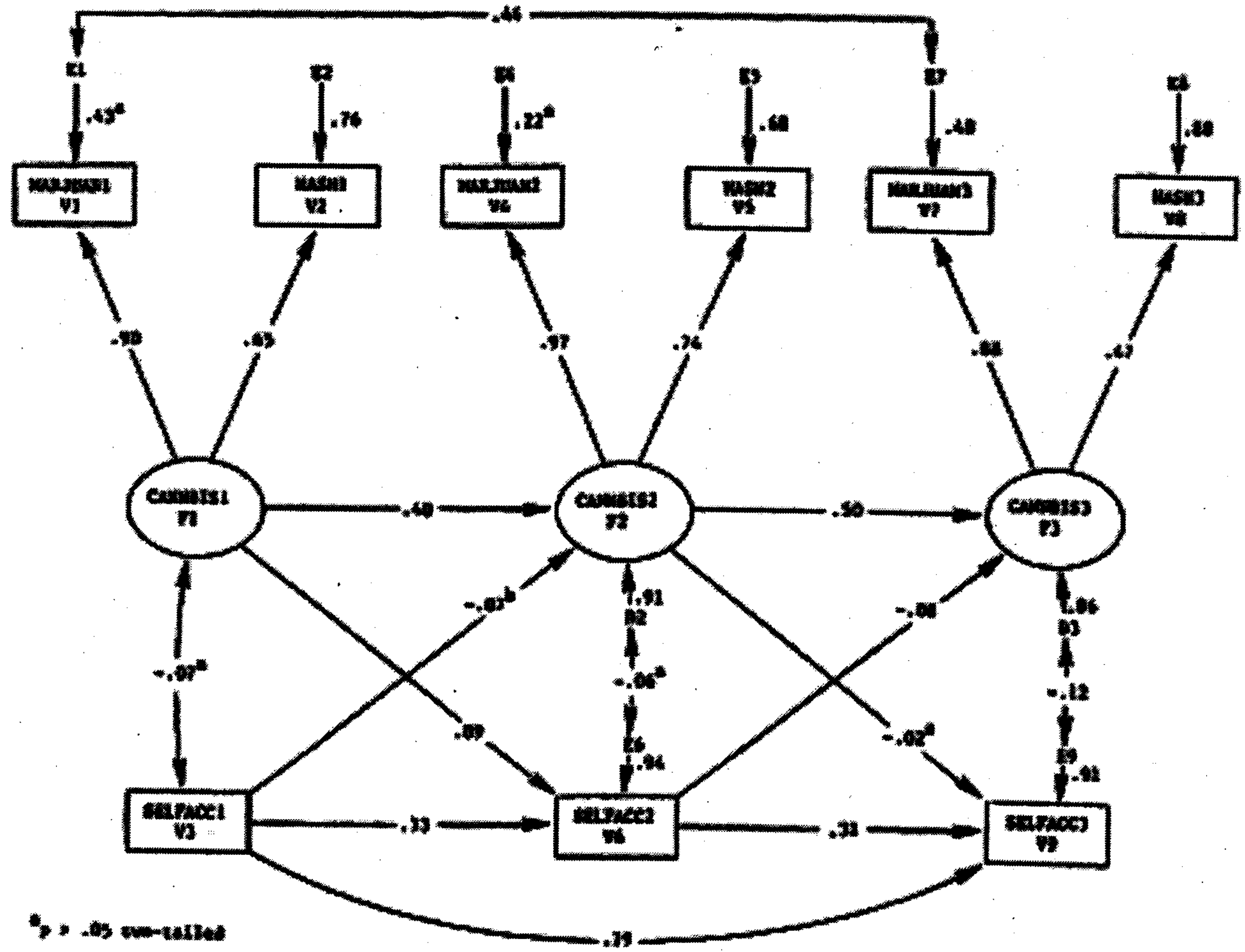


Fig. 2. Structural equations model. Large circles represent latent factors, and small circles with numbers reflect residual variances.



Bentler, P. M. (1987). Drug use and personality in adolescence and young adulthood: Structural models with nonnormal variables. *Child Development*, 58, 65-79.



$^a p > .05$ two-tailed

$^b p < .05$ one-tailed

FIG. 2.—Final solution for the model of Table 1 and Figure 1. Each variable has been standardized to unit variance, so that unidirectional arrows represent standardized beta coefficients, while two-way arrows represent correlations. All estimates are significant by univariate two-tailed tests, except for those marked with an *a* superscript. The regression coefficient marked by *b* is significant by a one-tail test.

Journal of Consulting and Clinical Psychology
1997, Vol. 65, No. 1, 130–140

0022-006X/97/$3.00

The Relation Between Adolescent Alcohol Use and Peer Alcohol Use: A Longitudinal Random Coefficients Model

Patrick J. Curran
Duke University

Eric Stice
Stanford University

Laurie Chassin
Arizona State University

Longitudinal latent growth models were used to examine the relation between changes in adolescent alcohol use and changes in peer alcohol use over a 3-year period in a community-based sample of 363 Hispanic and Caucasian adolescents. Both adolescent alcohol use and peer alcohol use were characterized by positive linear growth over time. Not only were changes in adolescent alcohol use closely related to changes in peer alcohol use, but the initial status on peer alcohol use was predictive of later increases in adolescent alcohol use and the initial status on adolescent alcohol use was predictive of later increases in peer alcohol use. These results are inconsistent with models positing solely unidirectional effects between adolescent alcohol use and peer alcohol use.

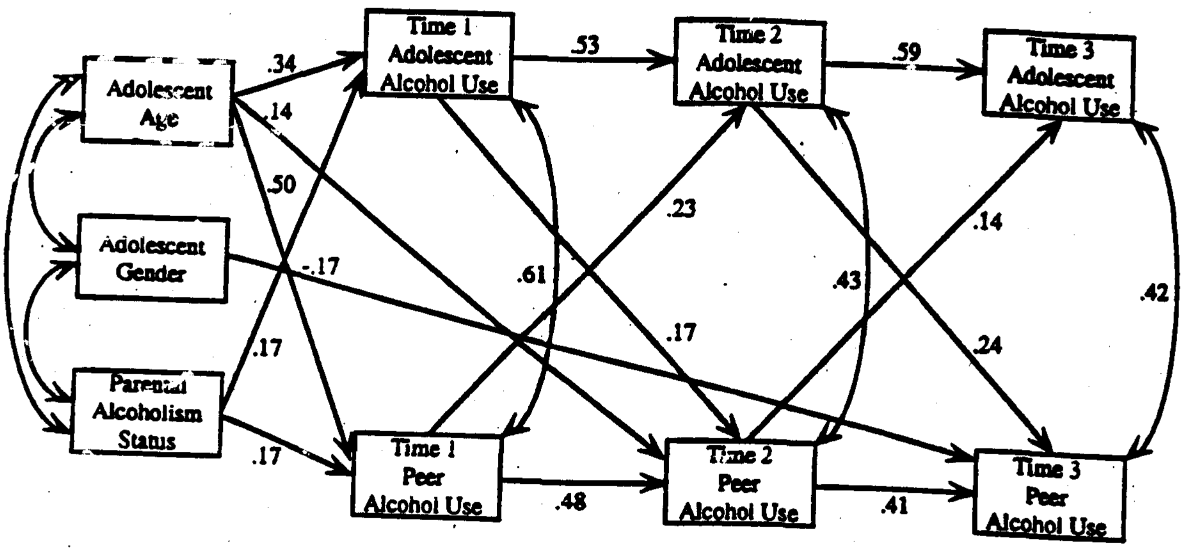
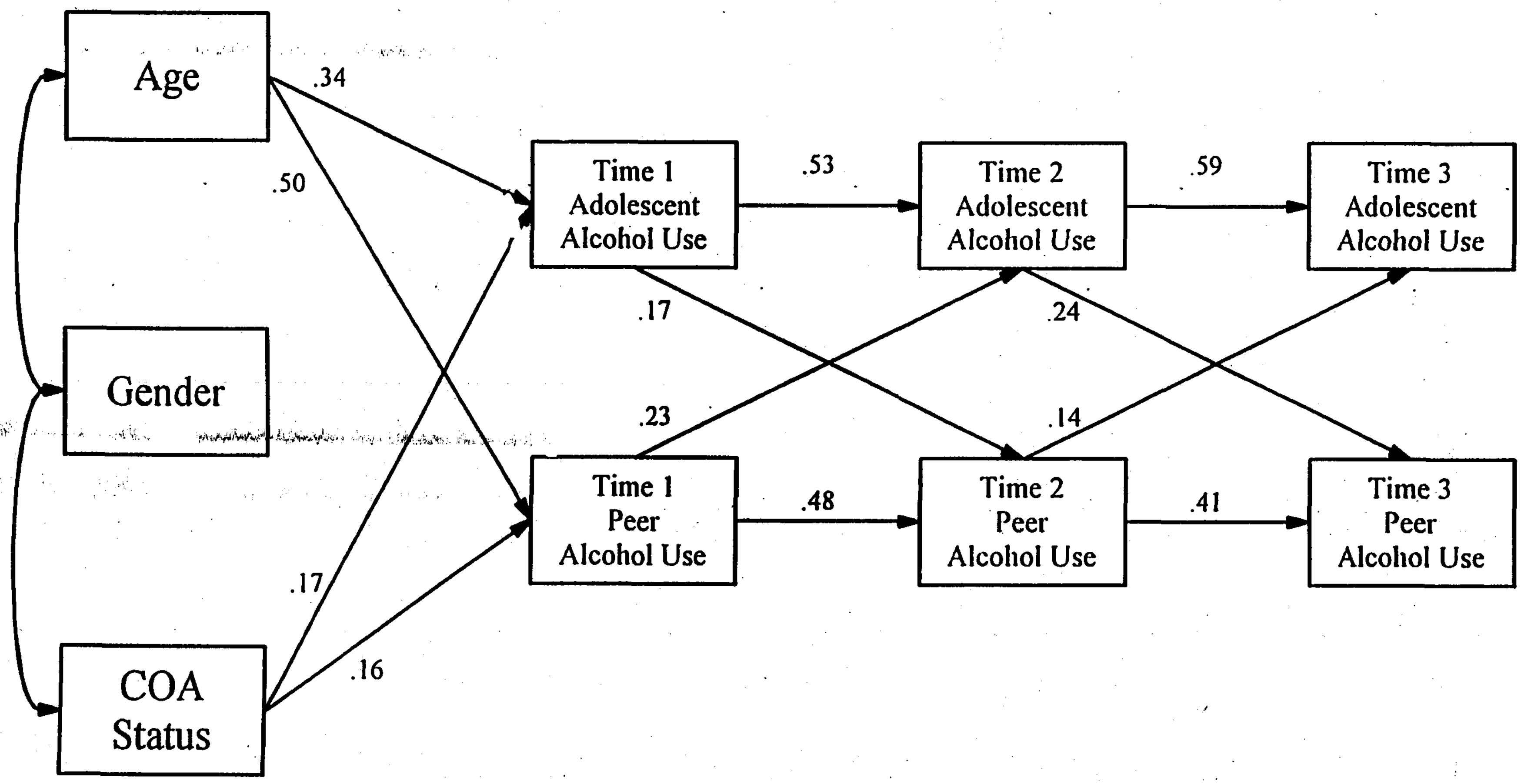


Figure 4. Autoregressive cross-lagged model of adolescent and peer alcohol use. Final model, $\chi^2(14, N = 363) = 29.3$, $p = .01$. All parameter values are standardized. All parameters shown are $p < .05$.



Chi-square (14, N=363)=29.2, p=.01, TLI=.97, CFI=.99

CURRAN et al.   JCCP

Mean and Covariance Structures

$$y = \alpha + \beta x + \varepsilon \qquad (1)$$

$$\mu_y = \alpha + \beta \mu_x, \qquad (2)$$

The parameters of any linear structural model with structured means are the regression coefficients, the variances and covariances of the independent variables, <u>the intercepts of the dependent variables, and the means of the independent variables.</u>

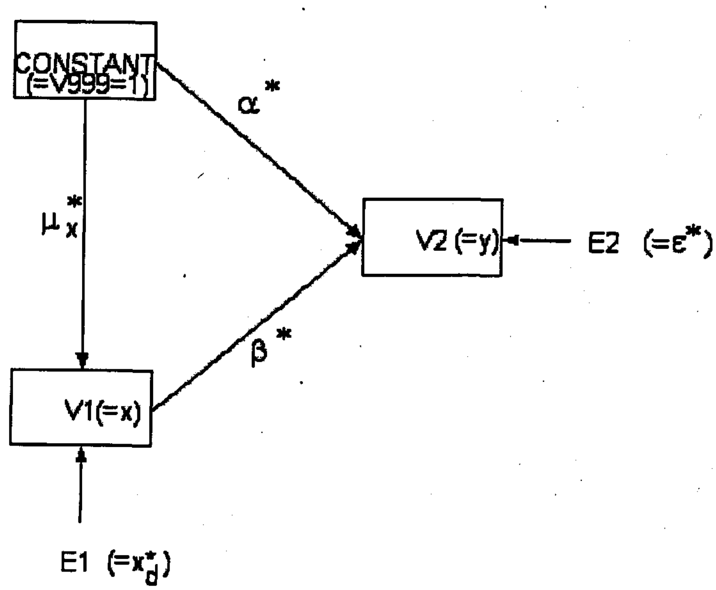
Rewrite (1)

$$y = \alpha 1 + \beta x + \varepsilon \qquad (3)$$

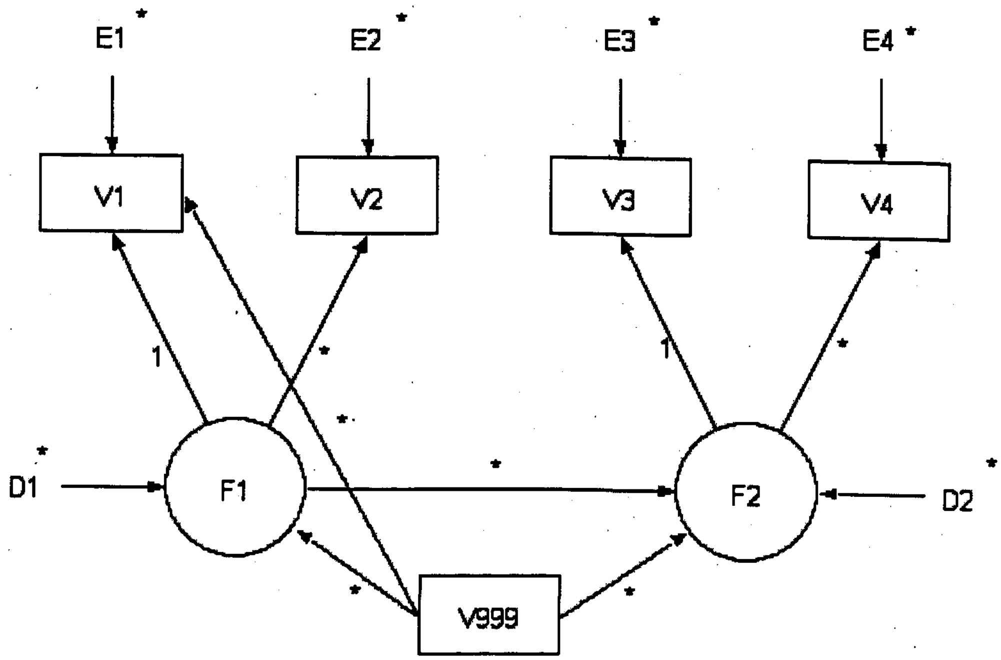
The coefficient for regression on a constant is an intercept.

Also, all measured and latent variables having nonzero means will be dependent variables.

$$x = \mu_x + x_d = \mu_x 1 + x_d, \qquad (4)$$



Structured Mean Example



In EQS, the constant 1 is V999 ("last variable in data file")

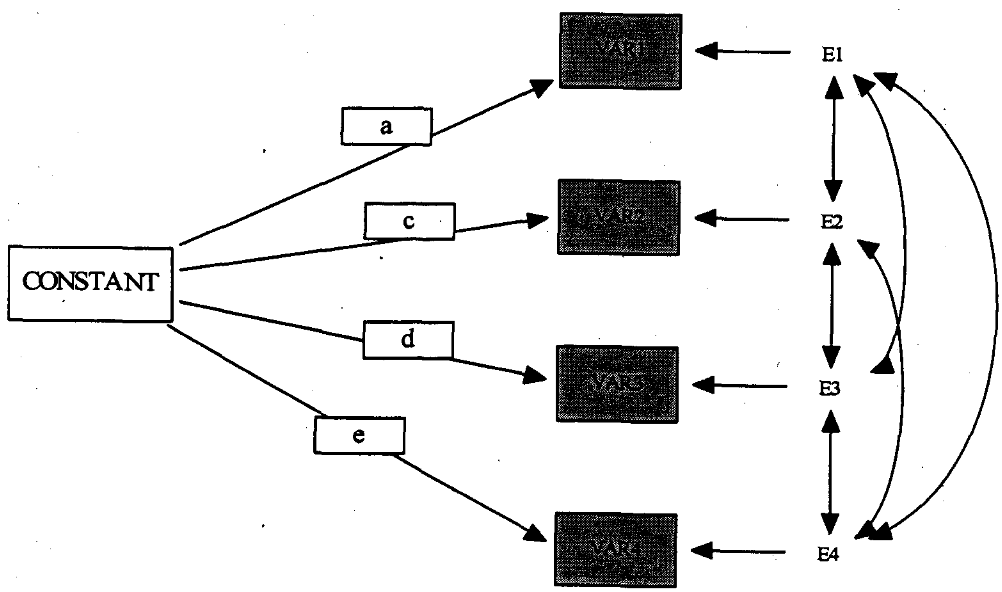
An intercept of a variable is its model-implied mean whenever there are no indirect effects of the constant (V999) on the variable. When there are such indirect effects, the total effect is the model-implied mean.

In standard SEM models, some parameters of covariance structure also are involved in mean structure, e.g., in the Bentler-Weeks approach used in EQS

$$\Sigma = G(I-B)^{-1}\Gamma\Phi\Gamma'(I-B)^{-1}{}'G'$$

$$\mu = G(I-B)^{-1}\Gamma\mu_{\xi}$$

However, mean structures can be completely unrelated to covariance structures – for example, we can have a linear growth structure on the means, and a completely unstructured covariance structure. Such separation of parameters is typical for anova, manova, time series, etc.



Above gives a linear mean growth with unstructured covariances. Note: c = a+b, where b is the linear increment; hence b=(c-a). Linear constraints on model are d=a+2(c-a); e=a+3(c-a).

Of course any other covariance structure might do on the right. Pan and Fan (2002) gave six, and Bentler (in J. Werner 2005) added two more

M_1: Σ is an arbitrary positive definite matrix (saturated model).

M_2: $\Sigma = \Lambda\Phi\Lambda' + \Lambda_c\Theta\Lambda_c'$

M_3: $\Sigma = \sigma^2\{(1-\rho)I_p + \rho 11'\}$

M_4: $\Sigma = \Lambda\Phi\Lambda' + \sigma^2 I_p$

M_5: $\Sigma = \sigma^2(\rho^{|i-j|})_{1\le i,j\le p}$

M_6: $\Sigma = \sigma^2 G$ for a given matrix G.

M_7: $\Sigma = \Lambda\Phi\Lambda' + \Psi$

M_8: $\Sigma = \Sigma(\theta)$.

M_1 applies if only interested in modeling mean growth.

M_2 is Rao's simple covariance structure, where the residual matrix is structured as the orthogonal complement Λ_c of the factor loading matrix Λ.

M_3 is the uniform or compound symmetry model of equal variances and equal covariances.

M_4 is called a random coefficient regression structure, but amounts to a factor model with equal residual variances which may be useful when sample size is small.

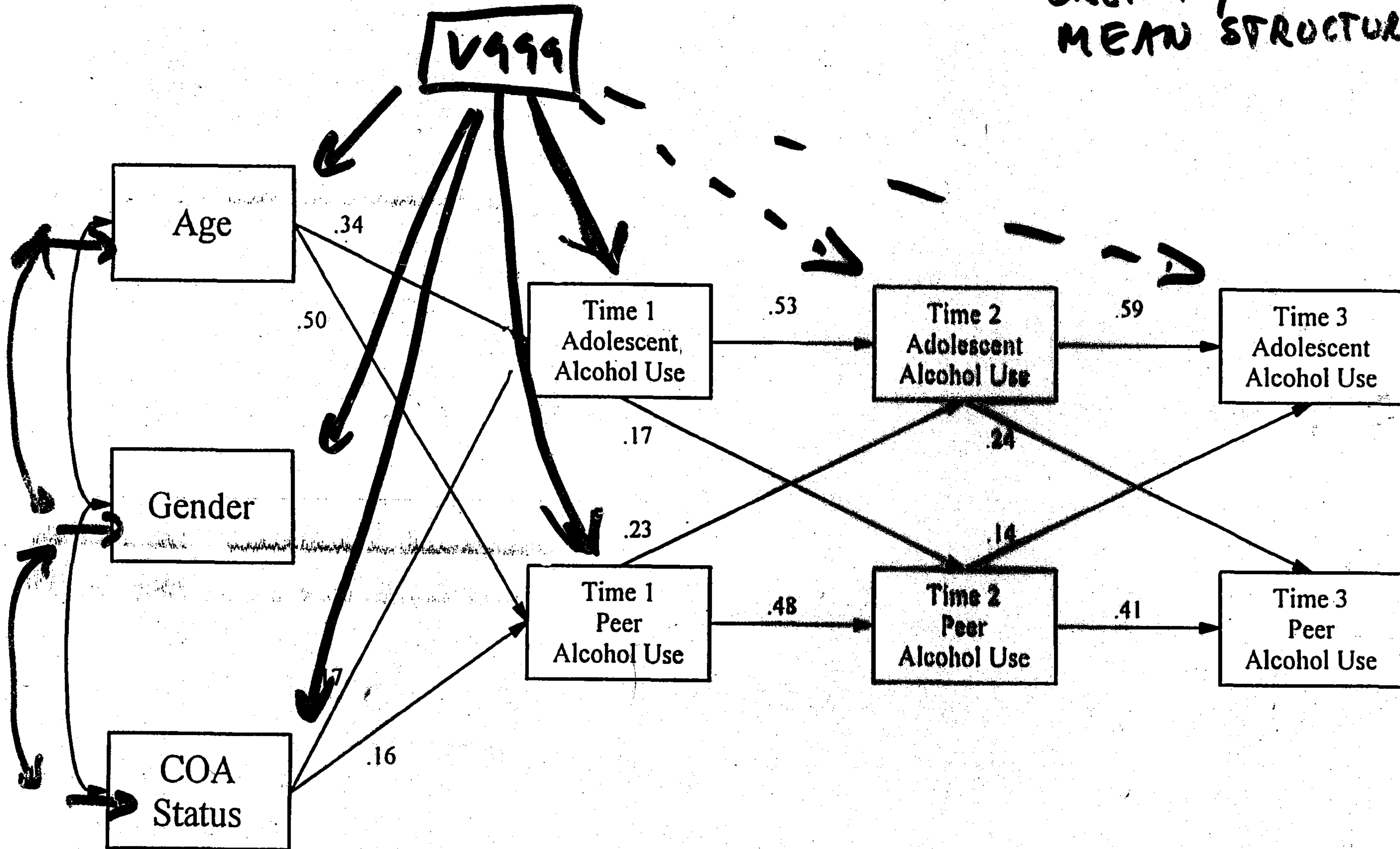
M_5 is a serial covariance structure.

M_6 allows the covariance matrix to be proportional to a known matrix.

M_7 is the usual confirmatory factor structure, the typical structure used in latent growth curve modeling.

M_8 states that any structure is possible.

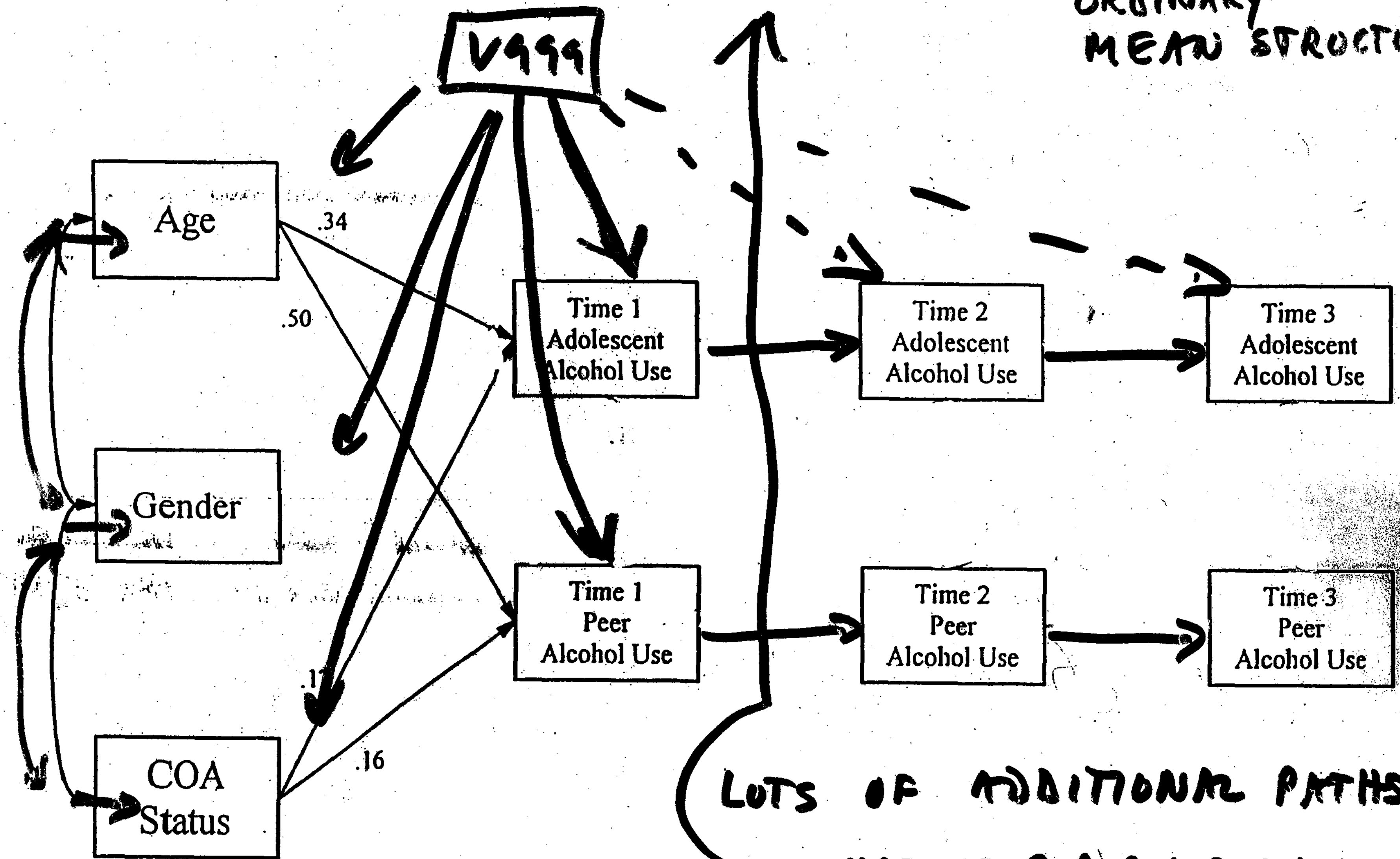
ORDINARY
MEAN STRUCTURES



Chi-square (14, N=363)=29.2, *p*=.01, TLI=.97, CFI=.99

CURRAN et al.   JCCP

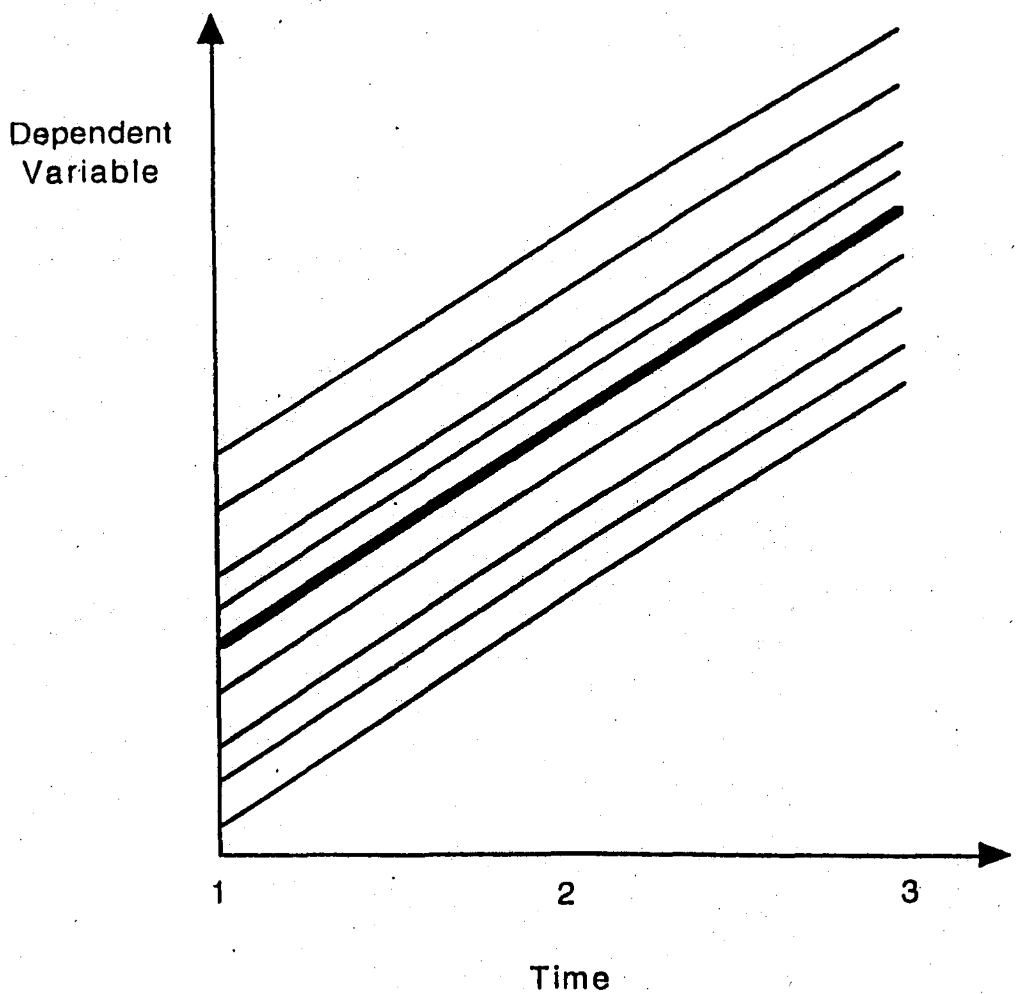
ORDINARY MEAN STRUCTURES



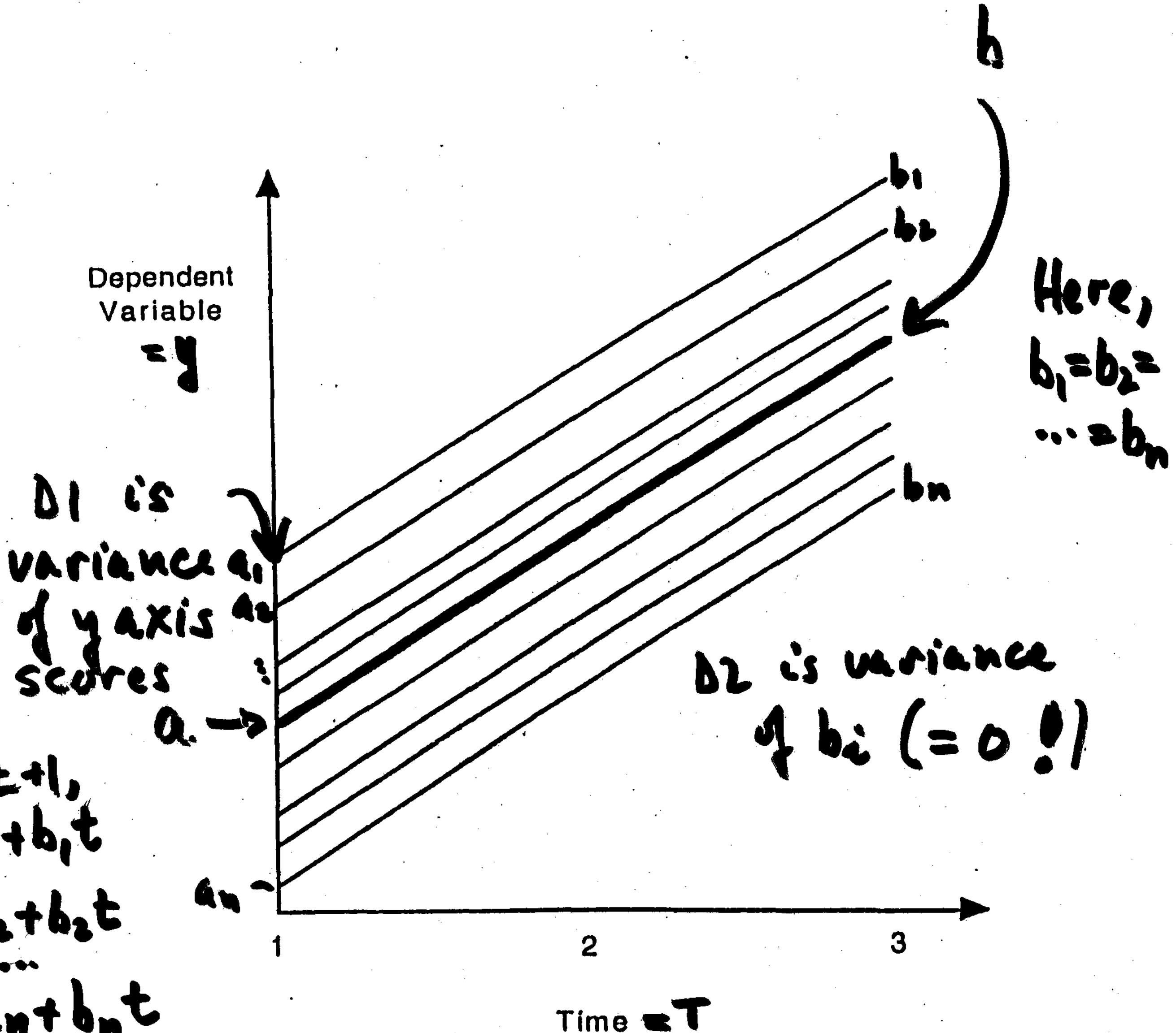
WHAT IF EXPERIMENTAL INTERVENTION OCCURRED HERE?

Chi-square (14, N=363)=29.2, p=.01, TLI=.97, CFI=.99

CURRAN et al. OR; CONTROL VS. EXPERIM. GROUPS



INDIVIDUAL TRENDS
+ GROWTH CURVE MODELS



At T=t+1,

$y_1 = a_1 + b_1 t$

$y_2 = a_2 + b_2 t$

...

$y_n = a_n + b_n t$

$\bar{y} = a + bt$

group parameters

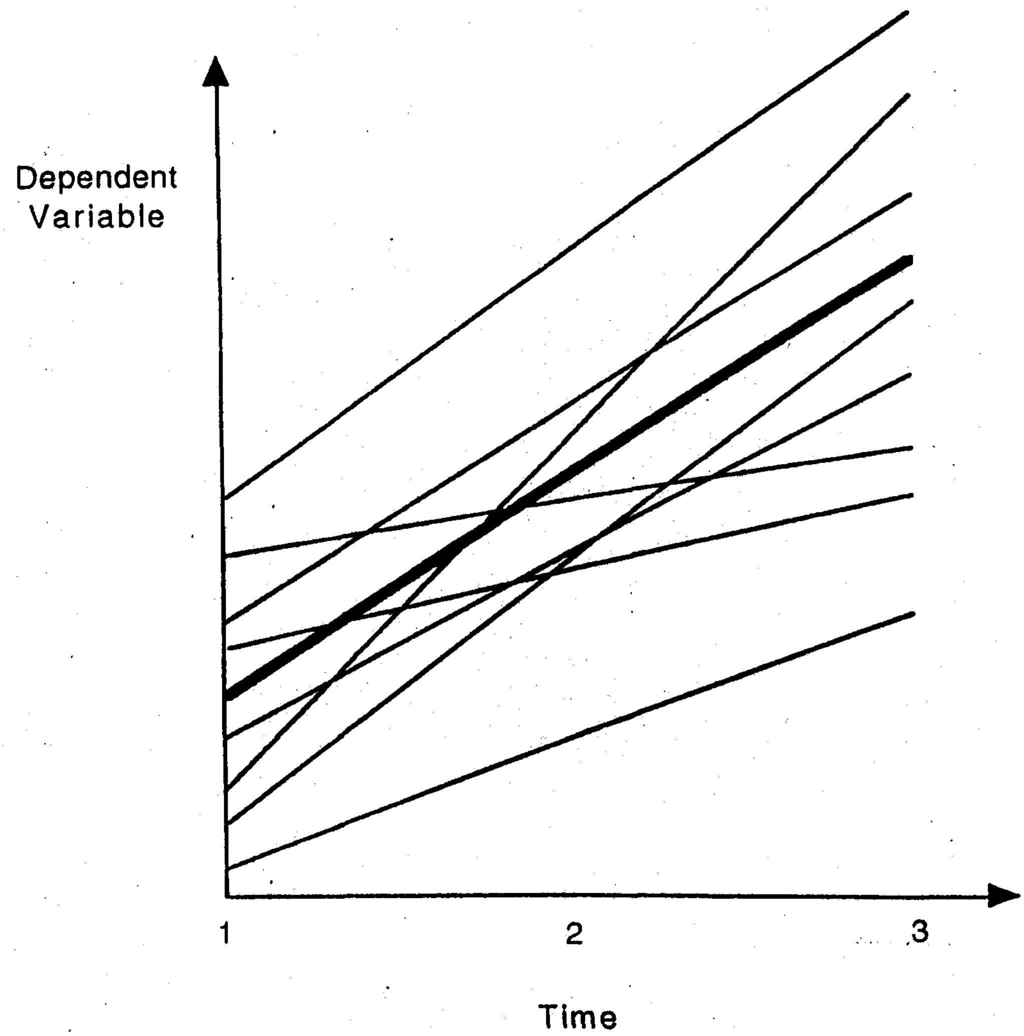
a = mean intercept

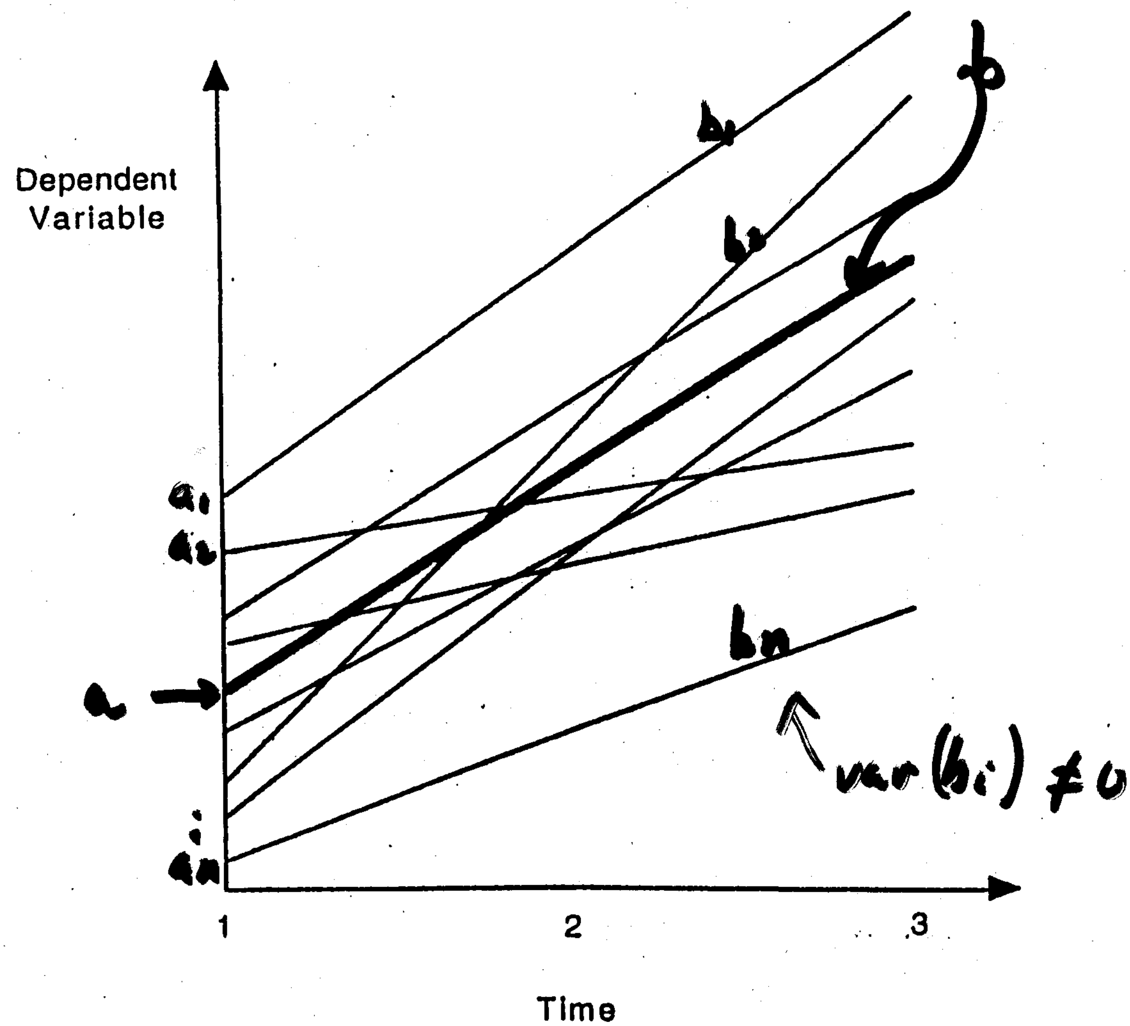
b = mean slope

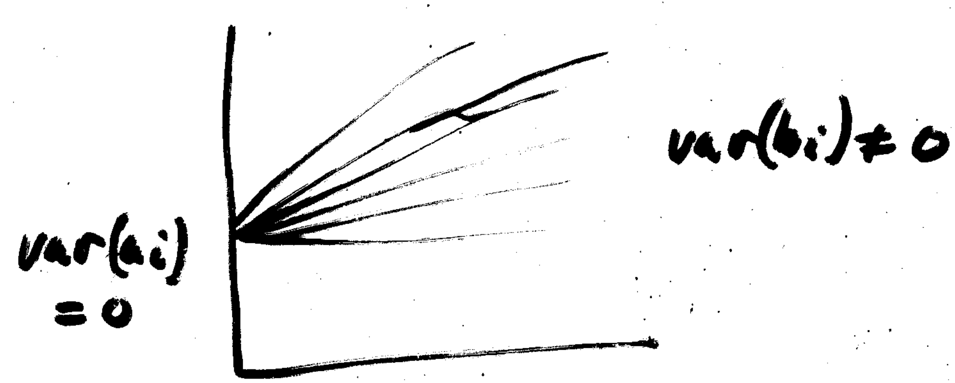
individual difference parameters

D1 = variance of intercepts ai

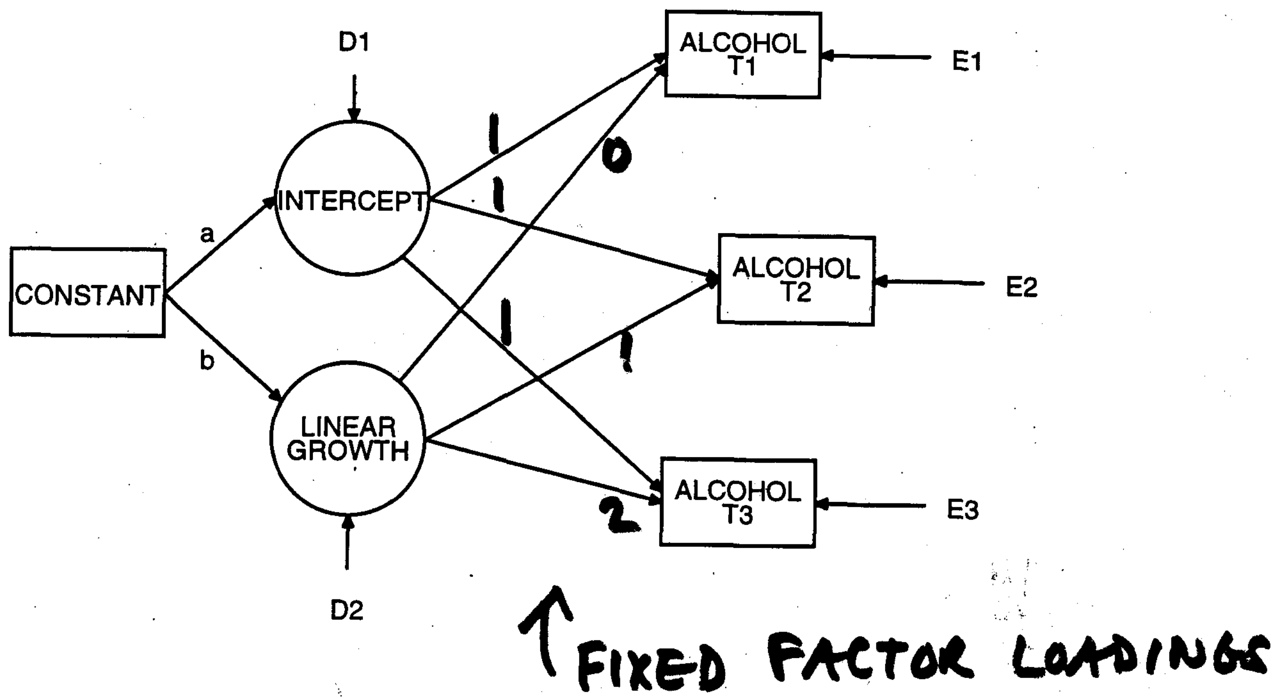
D2 = variance of slopes bi


Dependent
Variable
1
2
3
Time


Dependent
Variable
Time
1
2
3
var(bi) ≠ 0


var(ai)
= 0
var(bi) ≠ 0

BASIC GROWTH CURVE MODEL ILLUSTRATED FOR ALCOHOL USE



NOTES:

a -- Mean Alcohol Use at T1

b -- Mean Increment in Alcohol Use from T1 to T2, and T2 to T3

D1 -- Individual Differences in Alcohol Use at T1

D2 -- Individual Differences in Linear Slope of Alcohol Use across T1, T2, T3

Model-Implied Covariance Matrix and Means For Basic Growth Curve Model

(e.g., Alcohol T1, T2, T3)

	T1	**T2**	**T3**
T1	var (D1) + var (E1)		
T2	var (D1)	var (D1) + var (D2) + var (E2)	
T3	var (D1)	var (D1) + 2var (D2)	var (D1) + 4var (D2) + var (E3)
Means	a	a + b	a + 2b

NOTES:

a – Path from Constant to Intercept = Mean Alcohol Use at T1

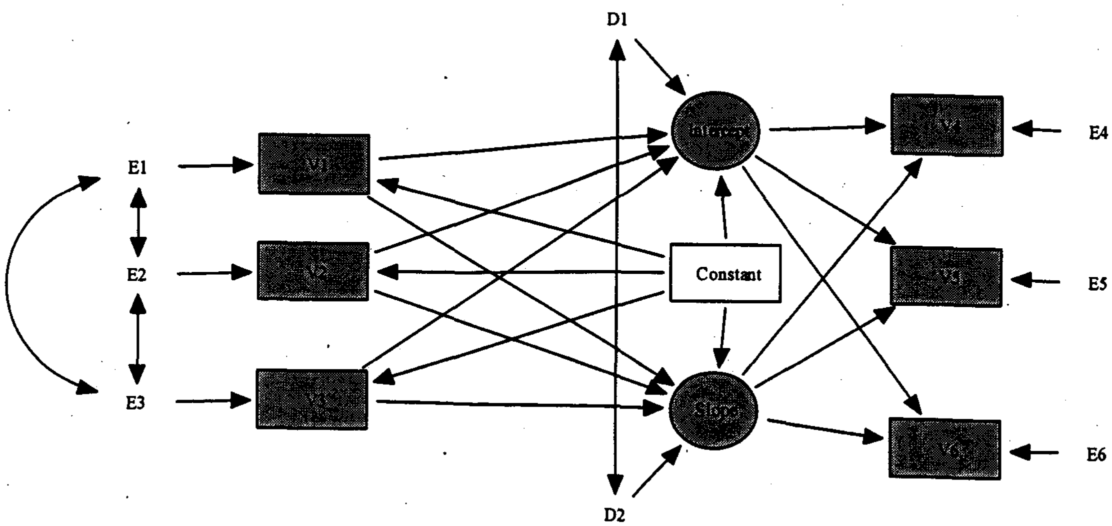
b – Path from Constant to Linear Growth = Increment in Means

LINEAR IN MEANS,

NOT VARIANCES

NOT COVARIANCES

Prediction of Linear Growth: EQS Setup*




```
/SPECIFICATIONS
VARIABLES=9; CASES=363;
METHOD=ML; ANALYSIS=MOMENT; MATRIX=COVARIANCE;
/EQUATIONS
V1 =    *V999 + E1;
V2 =    *V999 + E2;
V3 =    *V999 + E3;
V4 =    1F1 + 0F2 + E4;
V5 =    1F1 + 1F2 + E5;
V6 =    1F1 + 2F2 + E6;
F1 =    *V999 + *V1 + *V2 + *V3 + D1;
F2 =    *V999 + *V1 + *V2 + *V3 + D2;
/VARIANCES
E1 TO E6 = *;
D1 TO D2 = *;
/COVARIANCES
E1 TO E3 =*
D2, D1 = *;
```

*Get EQS from sales@mvsoft.com (free, works ~4 weeks)

Some Growth Curve Factor Loading Matrices Λ

Initial Status Model

Variable or Time	Intercept F1	Slope F2	Quadratic F3	Cubic F4	Quartic F5
V1	1	0	0	0	0
V2	1	1	1	1	1
V3	1	2	4	8	16
V4	1	3	9	27	81
V5	1	4	16	64	256
V6	1	5	25	125	625
V7	1	6	36	216	1296

The mean structure is $\mu = \Lambda\mu_\xi$

Interpretation of the Mean Structure (read each row)

Variable Mean	Intercept F1	Slope F2	Quadratic F3	Cubic F4	Quartic F5
μ_1 =	$1\mu_{F1}$	0	0	0	0
μ_2 =	$1\mu_{F1}+$	$1\mu_{F2}+$	$1\mu_{F3}+$	$1\mu_{F4}+$	$1\mu_{F5}$
μ_3 =	$1\mu_{F1}+$	$2\mu_{F2}+$	$4\mu_{F3}+$	$8\mu_{F4}+$	$16\mu_{F5}$
μ_4 =	$1\mu_{F1}+$	$3\mu_{F2}+$	$9\mu_{F3}+$	$27\mu_{F4}+$	$81\mu_{F5}$
μ_5 =	$1\mu_{F1}+$	$4\mu_{F2}+$	$16\mu_{F3}+$	$64\mu_{F4}+$	$256\mu_{F5}$
μ_6 =	$1\mu_{F1}+$	$5\mu_{F2}+$	$25\mu_{F3}+$	$125\mu_{F4}+$	$625\mu_{F5}$
μ_7 =	$1\mu_{F1}+$	$6\mu_{F2}+$	$36\mu_{F3}+$	$216\mu_{F4}+$	$1296\mu_{F5}$

Arbitrariness of the Scale of Time

$z = \Lambda\xi + \varepsilon$

$z = \Lambda T T^{-1}\xi + \varepsilon = \Lambda * \xi * + \varepsilon$

Hence the mean structure is arbitrary:

$\mu = \Lambda * \mu_\xi *$ where $\mu_\xi * = T^{-1}\mu_\xi$

The covariance structure is arbitrary too:

$\Sigma = \Lambda\Phi\Lambda' + \Psi$

$\Sigma = \Lambda * \Phi * \Lambda' * + \Psi$

$\Phi* = T^{-1}\Phi T^{-1'}$

In other words, the covariance or correlation between the factors depends on the chosen time scale. "The" correlation between initial status and growth (linear slope), for example, does not exist.

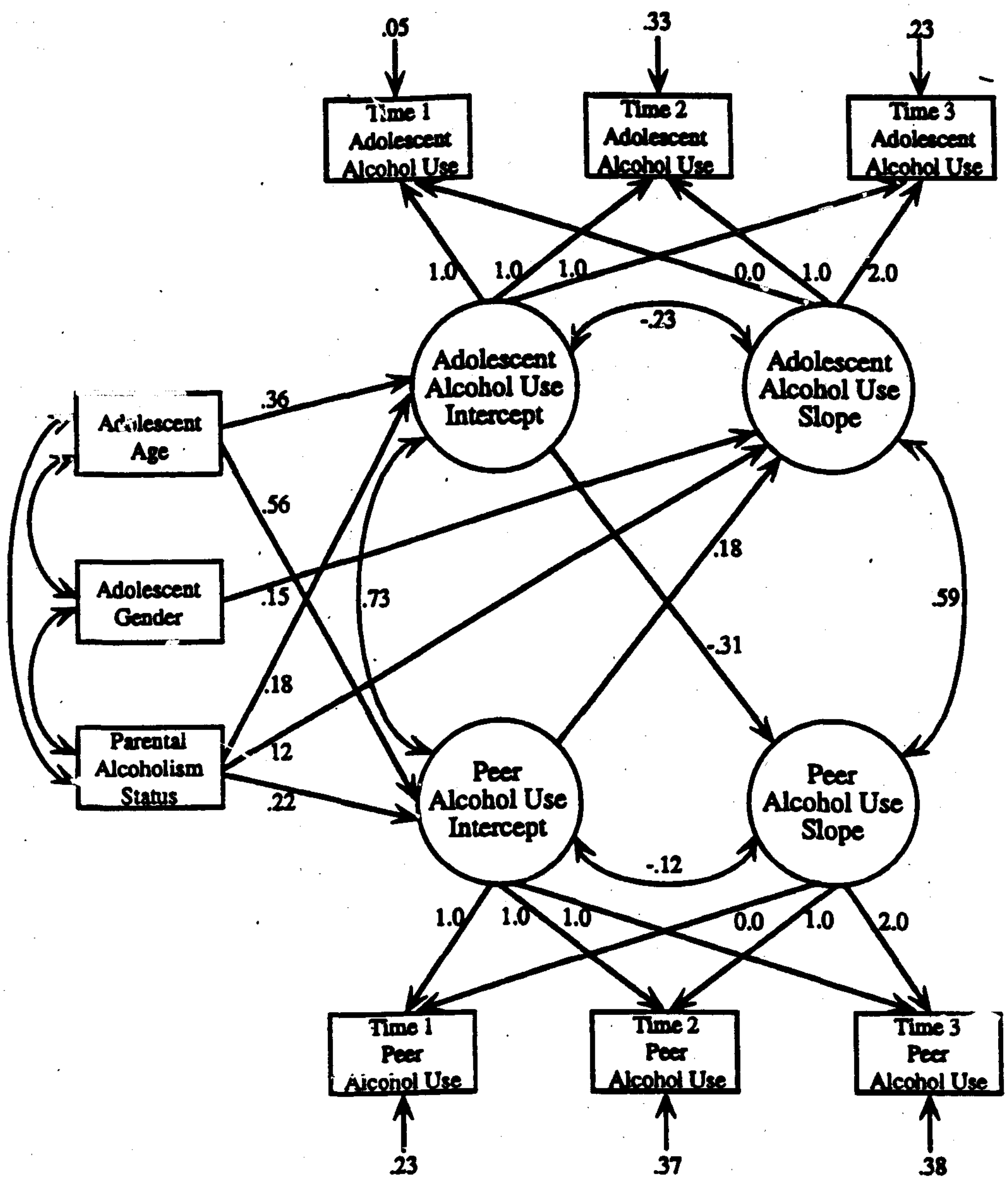
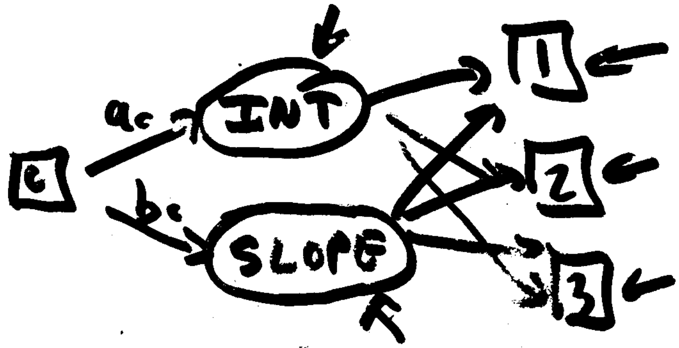


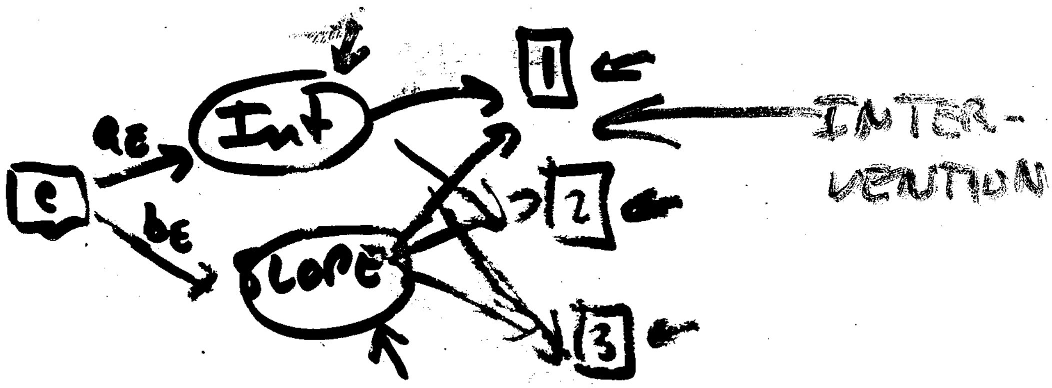
Figure 2. Final latent growth model of adolescent and peer alcohol use. Final model, $\chi^2(14, N = 363) = 25.4$, $p = .03$. All parameter values are standardized with the exception of the fixed factor loadings. All parameters shown are $p < .05$.

DELAY OF ONSET DUE TO PROGRAM INTERVENTION

CONTROL



EXPERIM.

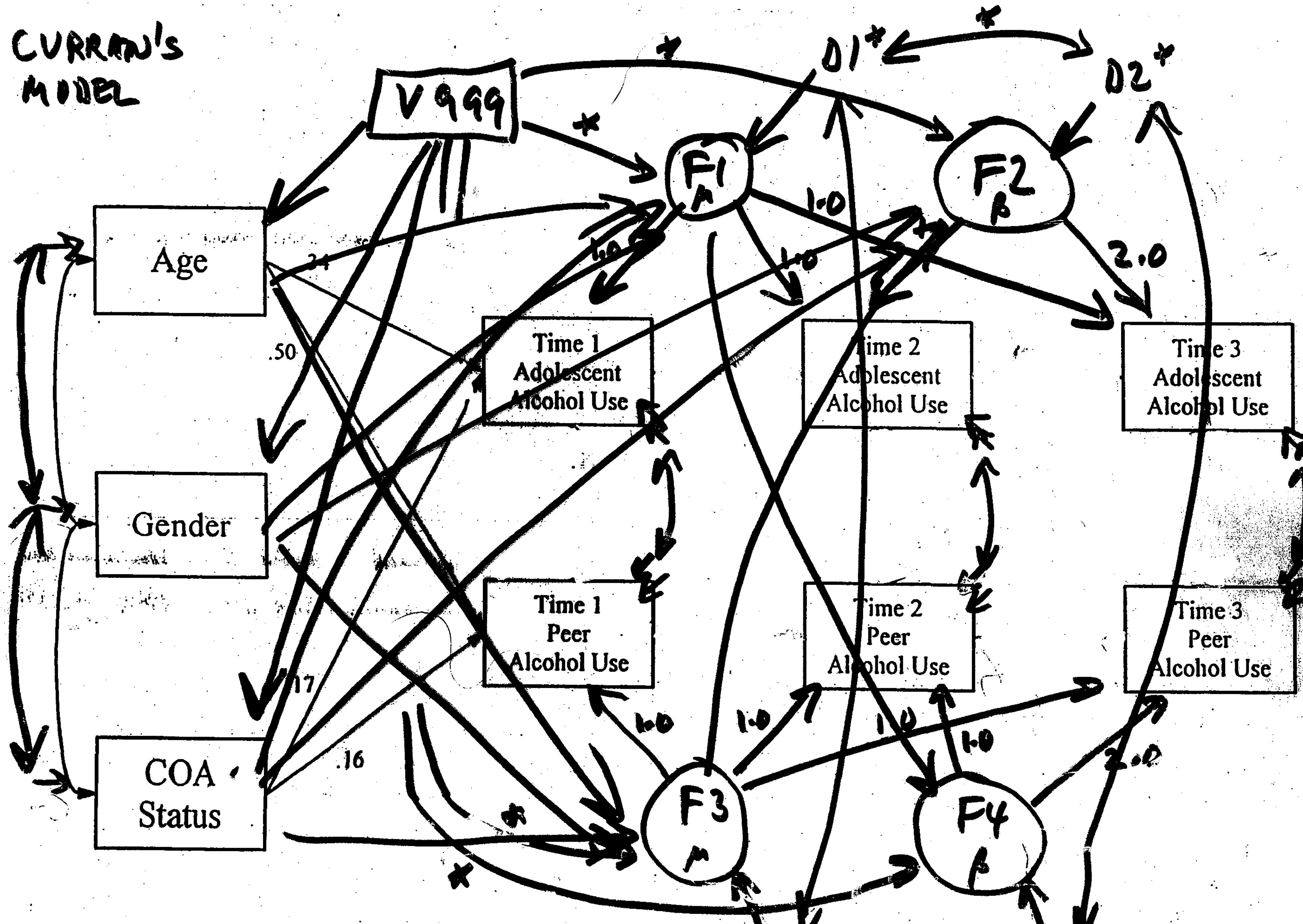


DELAY $\Rightarrow b_E < b_c$

SHOULD HAVE $a_E = a_c$

BUT IT'S NOT NECESSARY

CURRAN'S MODEL



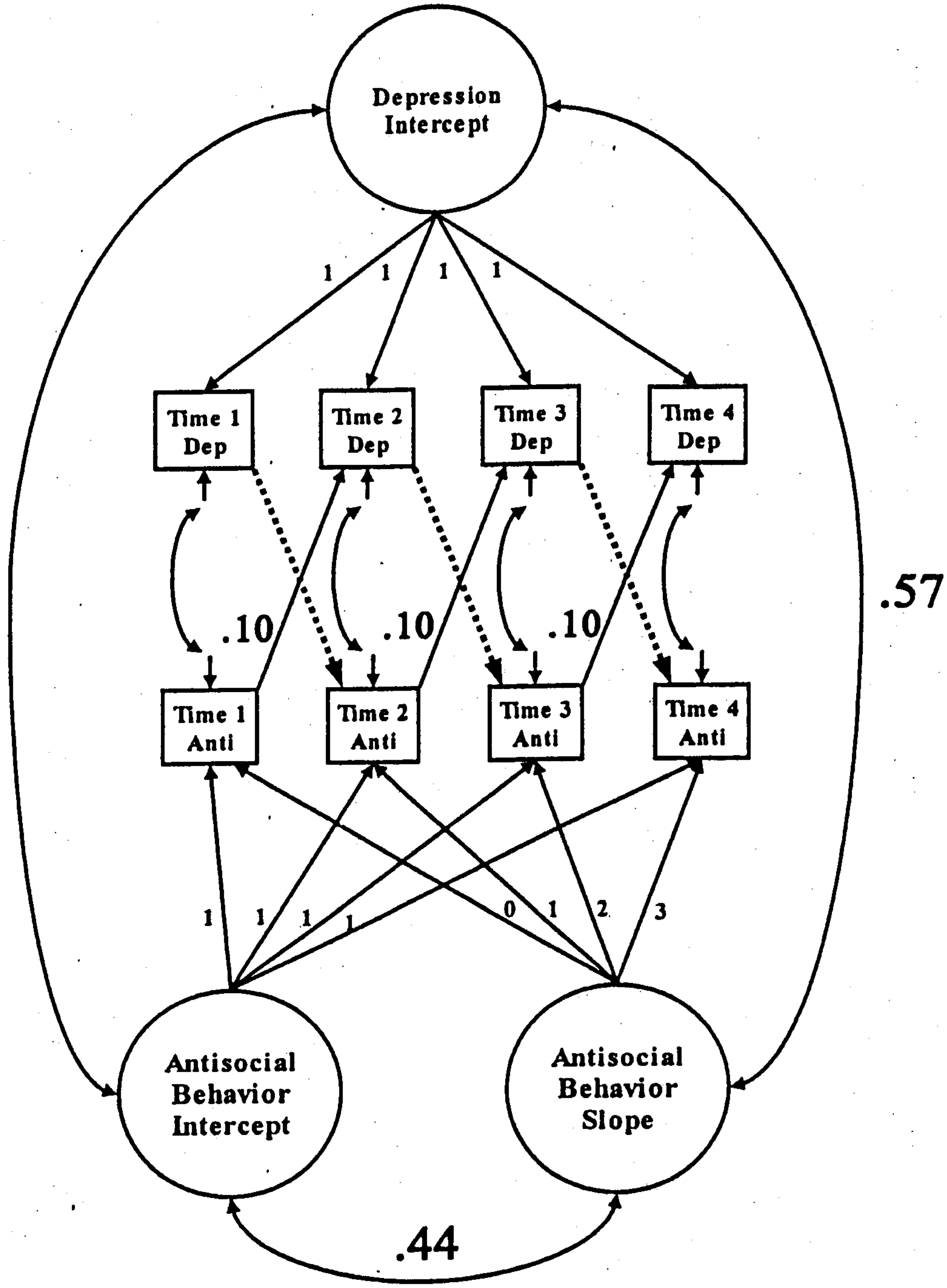
Chi-square (14, N=363)=29.2, p=.01, TLI=.97, CFI=.99

SOME ADDITIONAL PATHS

CURRAN et al.

FIGURE 4.8

Bivariate simplex latent curve model including lagged effects between indicators. Dep = depression; Anti = antisocial behavior.



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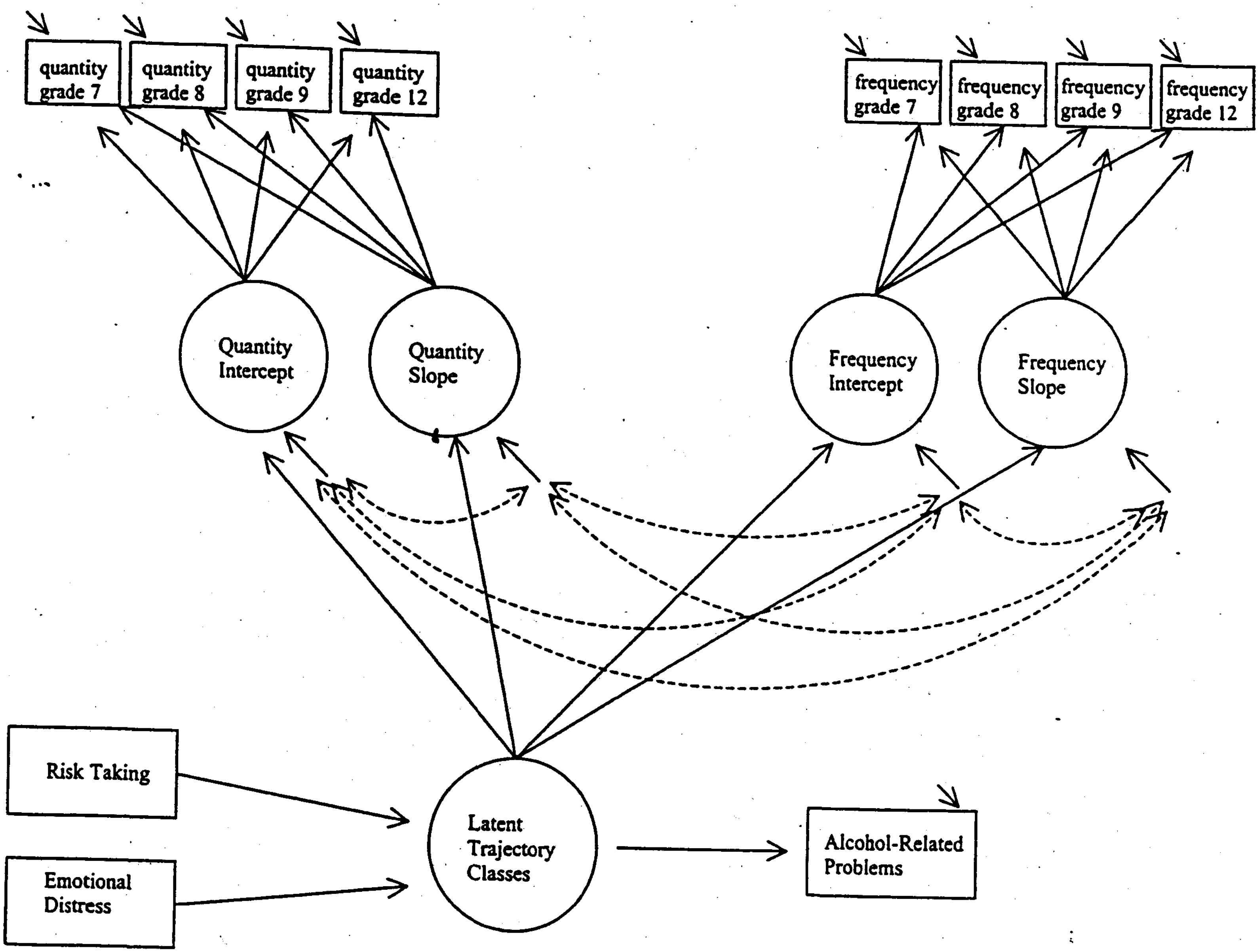

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